

Methods of Tacheometry:- There are two methods.

- 1) Stadia Tacheometry
- 2) TAO
- 3) Tangential Tacheometry.

1) Stadia Tacheometry:- In this method, the stadia diaphragm of the tacheometer is provided with two stadia hairs one upper hair & another lower hair, looking through the telescope the stadia hair readings are taken and noted. The difference between upper hair and lower hair readings is known as staff intercept [S]. To determine the distance between instrument station and staff, the staff intercept is multiplied by the multiplying constant [f/i]. The stadia method is further divided into two types.

- (a) Fixed Hair Method
- (b) Movable Hair Method

(a) Fixed Hair Method:- This is most commonly used method of tacheometry. In this Method, the stadia hairs are kept at fixed interval and the value of intercept ~~varies~~ varies with the distance from instrument station.

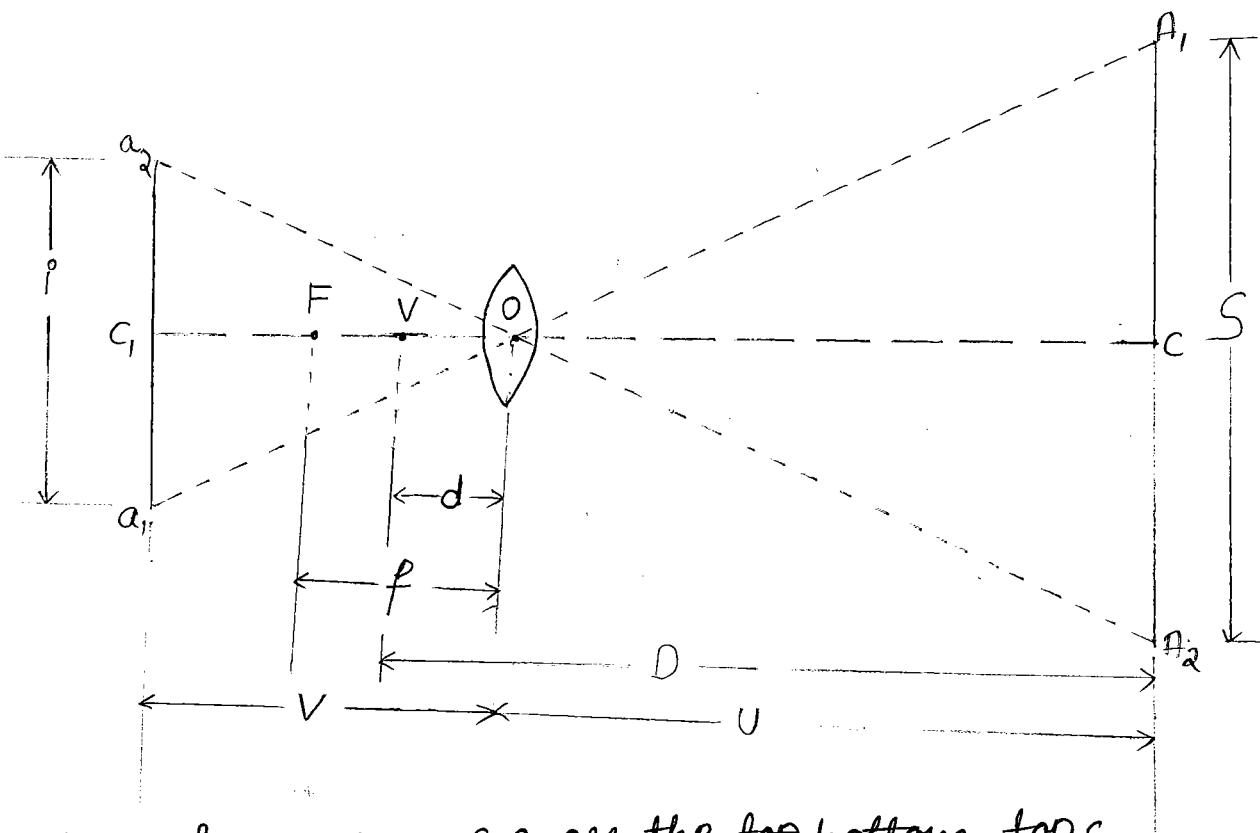
It contains of two cases.

Case-1: When line of sight is horizontal & staff held vertical

Case-2: When line of sight is inclined & staff held vertical

- (a) By considering angle of elevation.
- (b) By considering angle of depression.

Theory of Stadia Tacheometry (or) Determination of horizontal distance between instrument & staff when the line of sight is horizontal & staff held vertical (or) Determination of General tacheometric equation:-



In the above figure, a_1, a_2 & c , are the ~~top~~ bottom, top & central hairs of diaphragm. A_1, A_2, C are readings on staff cut by three hairs.

$a_1, a_2 = i$ = Stadia Intercept (or) length of image.

$A_1, A_2 = S$ = Staff intercept.

O = Optical centre of object glass

F = focus

V = Vertical axis of instrument.

f = focal length of object glass

d = Distance between optical centre & vertical axis of instrument.

u = Distance between optical centre & staff

v = Distance between optical centre & image

D = Horizontal distance between vertical axis of instrument & staff.

$$\therefore D = u + d \rightarrow ①$$

From triangles $O\alpha_1\alpha_2$, $O\alpha_2\alpha_1$, OA_1A_2 all similar triangles

\therefore from property of similar triangles,

$$\frac{i}{s} = \frac{v}{u} \Rightarrow v = \frac{iu}{s} \rightarrow ②$$

From property of lenses, we know that

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow ③$$

By substituting eq ② in eq ③, we get

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{\frac{iu}{s}}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{s}{ui}$$

$$\frac{1}{f} = \frac{1}{u} \left[1 + \frac{s}{i} \right]$$

$$u = f \left[1 + \frac{s}{i} \right]$$

but from eq ①,

$$D = u + d$$

$$D = f \left(1 + \frac{s}{f} \right) + d$$

$$D = f + \frac{f(s)}{f} + d$$

$$D = \frac{f}{f} (s) + [f + d] \rightarrow ④$$

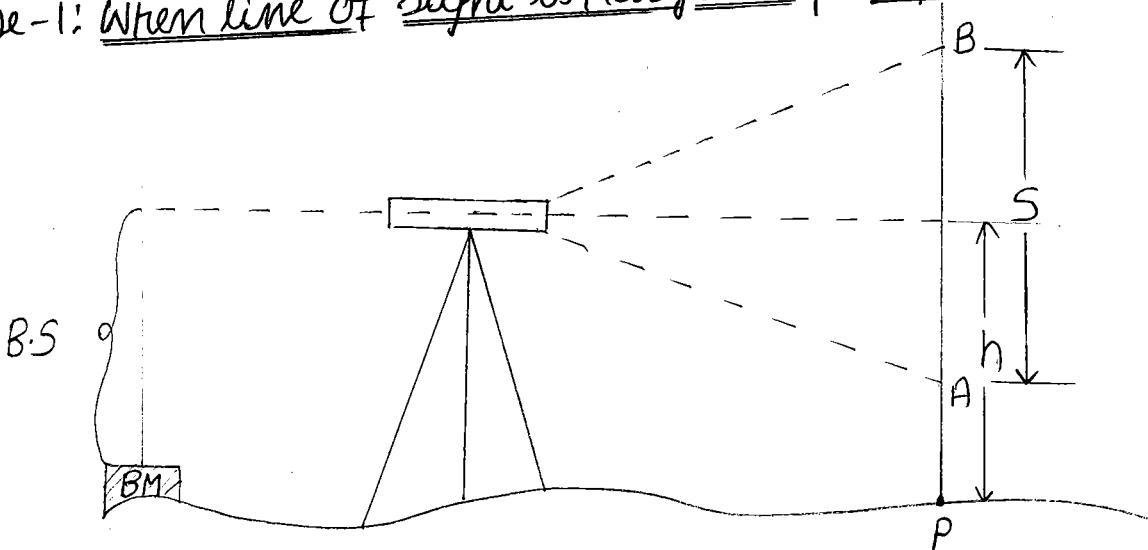
Equation ④ is General tacheometric equation.

The, $\left[\frac{f}{f} \right]$ is multiplying constant & $(f+d)$ = Additive constant

$\left[\frac{f}{f} \right]$ & $(f+d)$ are tacheometric constants

Fixed Hair Method:-

case-1: When line of sight is horizontal & staff held vertical:



h = central hair Reading

When the line of sight is horizontal & staff held vertical,
The General tacheometric equation is for Distance,

$$D = \frac{f}{f} (s) + [f + d]$$

RL of instrument =

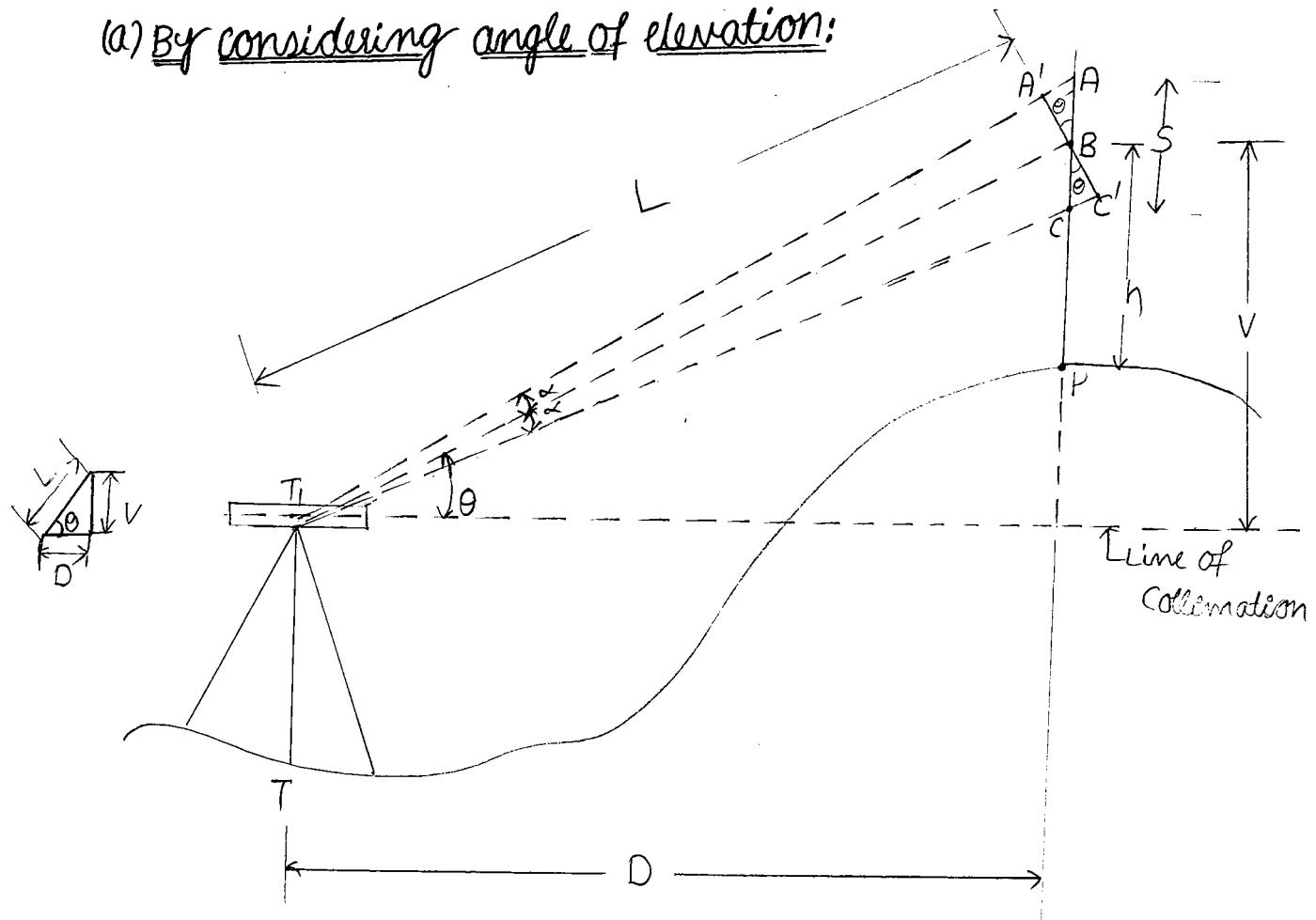
$$\text{RL of Height of Instrument} = \text{RL of B.M} + \text{B.S}$$

$$\text{RL of staff station A} = \text{RL of Height of Instrument} - h \\ (\text{Q})$$

$$\text{RL of staff station A} = \text{RL of BM} + \text{B.S} - h$$

case - 2: When line of sight is inclined & staff held vertical:

(a) By considering angle of elevation:



In the above figure,

T = instrument station

P = staff station

AC = S = staff intercept [upper Hair Reading -
Lower Hair Reading]

h = central hair reading

V = vertical distance of central hair ~~reading~~ above line of collimation.

D = Horizontal distance b/w Instrument & staff.

$T, B = L$ = inclined distance

θ = Angle of elevation [It is an angle between central hair & line of collimation)

α = Angle made by upper & lower stadia rays with central rays.

A, B, C = staff readings cut by stadia hairs

Let, $A'C'$ is drawn perpendicular to $T'B$.

Now inclined distance $T, B[L]$

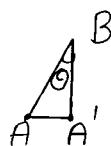
$$= \left[\frac{f}{i} \right] A'C' + [f+d] \rightarrow ①$$

Horizontal distance, $D = l \cos \theta$

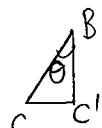
By substituting the value of ' L ' from eq ①

$$D = \left[\frac{f}{i} \right] A'C' \cos \theta + [f+d] \cos \theta \rightarrow ②$$

$$A'C' = A'B + BC'$$



$$\cos \theta = \frac{A'B}{AB} \Rightarrow AB \cos \theta = A'B$$



$$\cos \theta = \frac{C'B}{BC} \Rightarrow BC \cos \theta = C'B$$

$$\begin{aligned} A'C' &\Rightarrow A'B + BC' \Rightarrow AB \cos \theta + BC \cos \theta. \quad [\because AB + BC = S] \\ &\Rightarrow \cos \theta (AB + BC) \\ &\Rightarrow \cos \theta (S) \Rightarrow S \cos \theta. \end{aligned}$$

By substitute A'C' value in eq ②, then

Horizontal distance, $D = \left[\frac{f}{i} \right] \cos \theta (\cos \theta) + [f + d] \cos \theta$

$$D = \left[\frac{f}{i} \right] \cos^2 \theta + [f + d] \cos \theta \rightarrow ③$$

from the figure

Vertical distance, $V = \cancel{L \sin \theta} L \sin \theta$

$$V = \left[\frac{f}{i} \right] A'C' + [f + d] \sin \theta$$

$$V = \left(\frac{f}{i} \right) \cos \theta \sin \theta + [f + d] \sin \theta$$

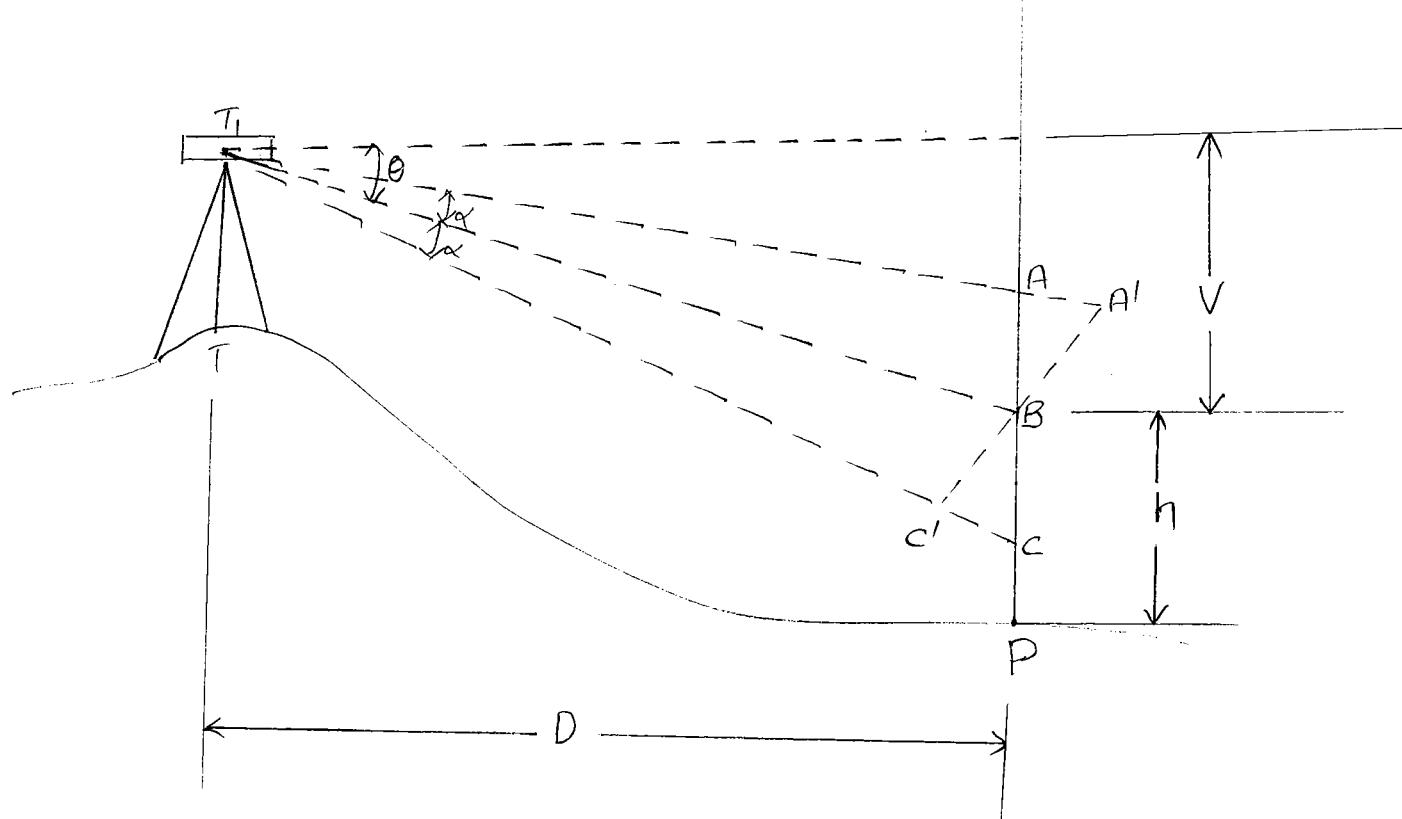
$$\left[\because \sin 2\theta = 2 \sin \theta \cos \theta \right]$$

$$\frac{\sin 2\theta}{2} = \sin \theta \cos \theta$$

$$V = \left(\frac{f}{i} \right) S \frac{\sin 2\theta}{2} + [f + d] \sin \theta \rightarrow ④$$

RL of P = RL of line of collimation + V - h

(b) By considering angle of depression



Horizontal distance,

$$D = \left[\frac{f}{f+d} \right] (s) \cos^2 \theta + [f+d] \cos \theta \rightarrow ①$$

Vertical distance,

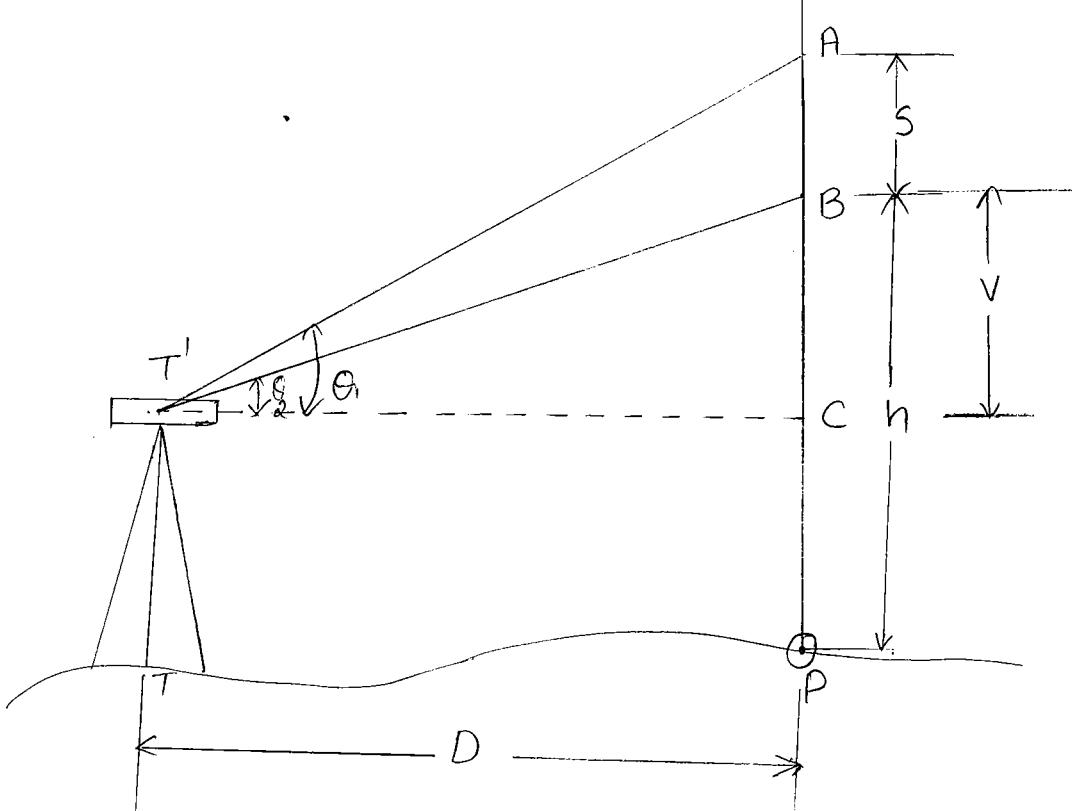
$$V = \left[\frac{f}{f+d} \right] (s) \frac{\sin 2\theta}{2} + [f+d] \sin \theta \rightarrow ②$$

RL of P = RL of line of collimation - V - h.

2) Tangential Tacheometry: This method is generally adopted when the theodolite is not provided with stadia diaphragm. In this method, the horizontal & vertical distances are calculated from the observed vertical angles to two points fixed at a known distance apart on the staff. This method is not preferred because atleast two vertical angles are required to be taken. But incase of tacheometry only one vertical angle is sufficient.

Depending upon the vertical angles ~~are~~ three cases are adopted

case-1: When both angles of target are angle of elevation:



In the above figure,

T = Instrument station

P = staff station

S = Distance between targets

V = Vertical distance b/w lower target & Line of collimation.

h = Height of lower target above the staff station.

θ_1 = vertical angle made by upper target

θ_2 = vertical angle made by lower target

From $\triangle AT'C$,

$$\tan \theta_1 = \frac{V+S}{D} \Rightarrow V+S = D \tan \theta_1 \rightarrow (1)$$

from $\triangle BT'C$,

$$\tan \theta_2 = \frac{V}{D} \Rightarrow D \tan \theta_2 = V \rightarrow (2)$$

Substitute eq (2) in eq (1), we get

$$D \tan \theta_2 + S = D \tan \theta_1$$

$$S = D \tan \theta_1 - D \tan \theta_2$$

$$S = D[\tan \theta_1 - \tan \theta_2]$$

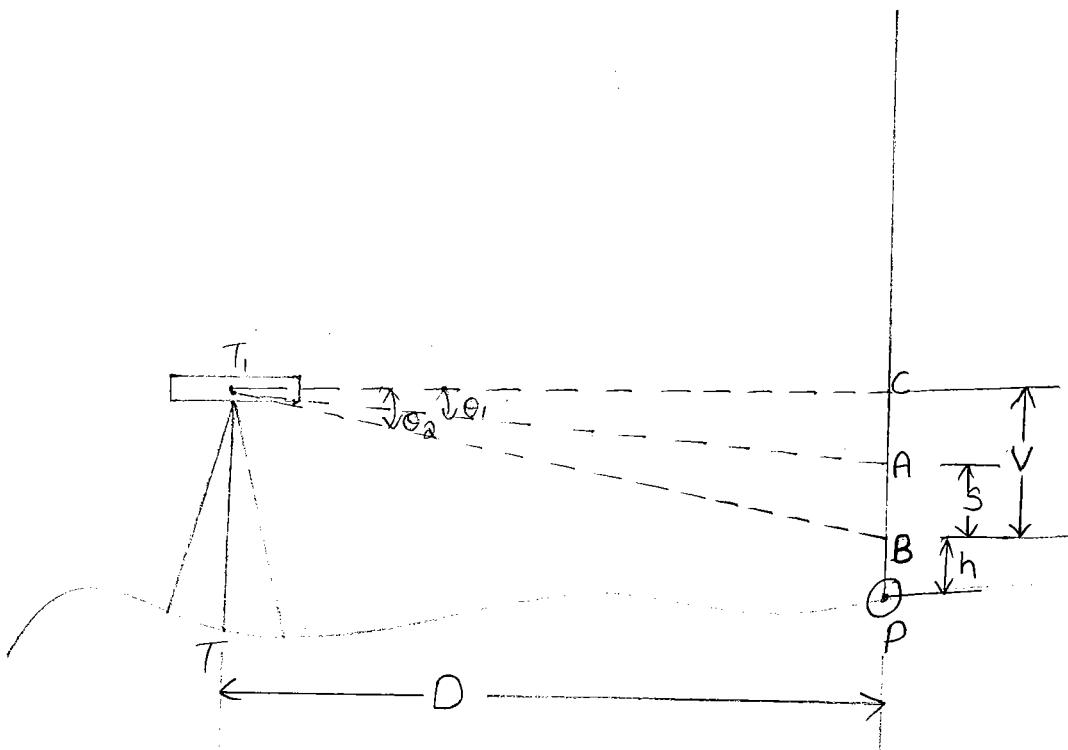
$$D = \frac{S}{[\tan \theta_1 - \tan \theta_2]} \rightarrow (3)$$

from eq (2),

$$V = D \tan \theta_2 \Rightarrow V = \frac{S \tan \theta_2}{[\tan \theta_1 - \tan \theta_2]} \rightarrow (4)$$

RL of P = RL of line collimation + V-h

case-2: When both angles of targets are angle of depression.



In the above figure,

$$\text{From } \triangle CT_1A, \tan \theta_1 = \frac{V-S}{D} \Rightarrow V-S = D \tan \theta_1 \rightarrow ①$$

$$\text{From } \triangle CT_2B, \tan \theta_2 = \frac{V}{D} \Rightarrow V = D \tan \theta_2 \rightarrow ②$$

Substitute eq ② in eq ①, then

$$D \tan \theta_2 - S = D \tan \theta_1$$

$$D \tan \theta_2 - D \tan \theta_1 = S$$

$$D [\tan \theta_2 - \tan \theta_1] = S$$

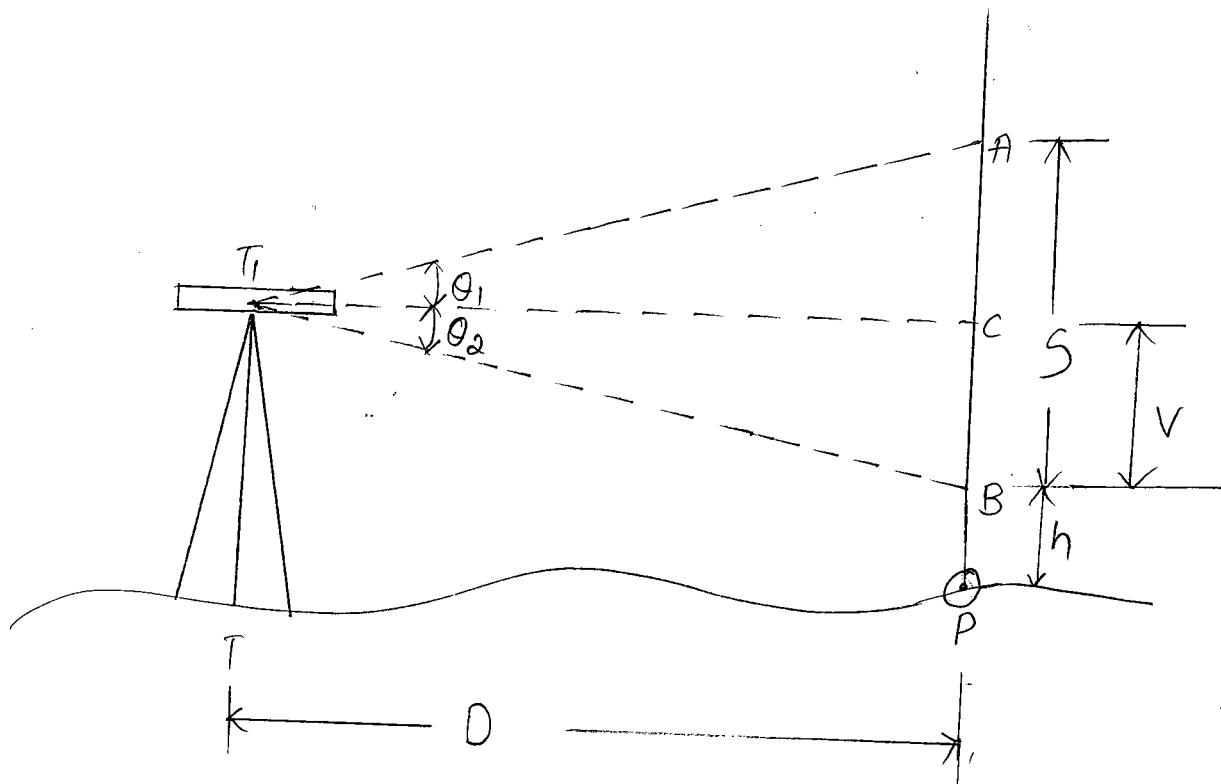
$$D = \frac{S}{[\tan \theta_2 - \tan \theta_1]} \rightarrow ③$$

from eq ②, $V = D \tan \theta_2$

$$V = \frac{S \tan \theta_2}{[\tan \theta_2 - \tan \theta_1]} \rightarrow ④$$

RL of P = RL of line of collimation - V-h.

CASE-3: When one angle is angle of elevation & other angle is angle of depression:



In the above figure,

From $\triangle ATC$,

$$\tan \theta_1 = \frac{S-V}{D} \rightarrow S-V \text{ } D \tan \theta_1,$$

$$D \tan \theta_1 = S-V \rightarrow ①$$

From $\triangle CTB$,

$$\tan \theta_2 = \frac{V}{D}$$

$$D \tan \theta_2 = V \rightarrow ②$$

By substituting the value of 'V' from ② in ①, we get

$$S - D \tan \theta_2 = D \tan \theta_1$$

$$S = [D \tan \theta_1 + D \tan \theta_2]$$

$$S = D [\tan \theta_1 + \tan \theta_2]$$

$$D = \frac{S}{[\tan \theta_1 + \tan \theta_2]} \rightarrow ③$$

from eq ②,

$$V = D \tan \theta_2$$

$$V = \frac{S \tan \theta_2}{[\tan \theta_1 + \tan \theta_2]} \rightarrow ④$$

$$\therefore \text{RL of P} = \text{RL of line of collimation} - V - h.$$

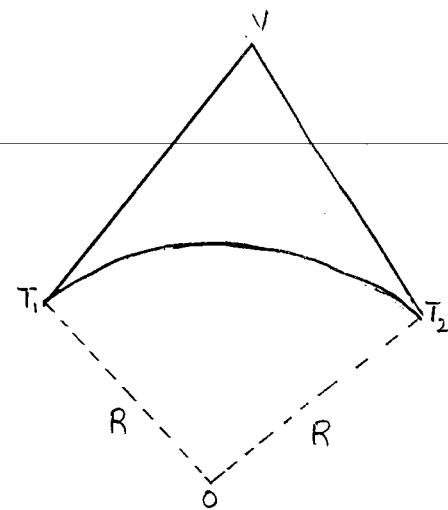
5. "CURVES"

* "A curve may be circular, parabolic or spiral and is always tangential to the two straight directions is called a curve."

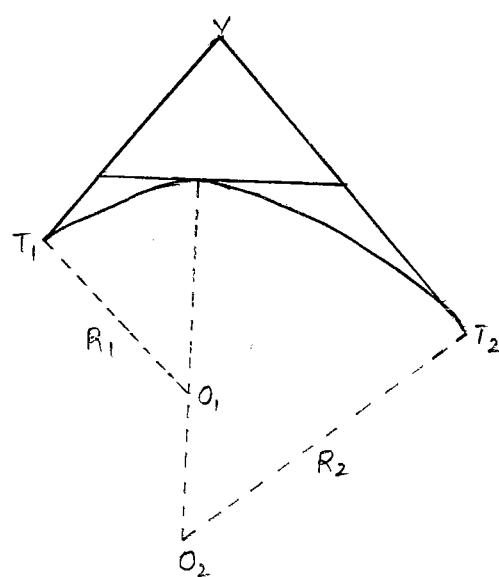
* Types of curves :- Curves are divided into ~~three~~ four types.

1. Simple
2. compound and
3. Reverse.
4. Transition curve.

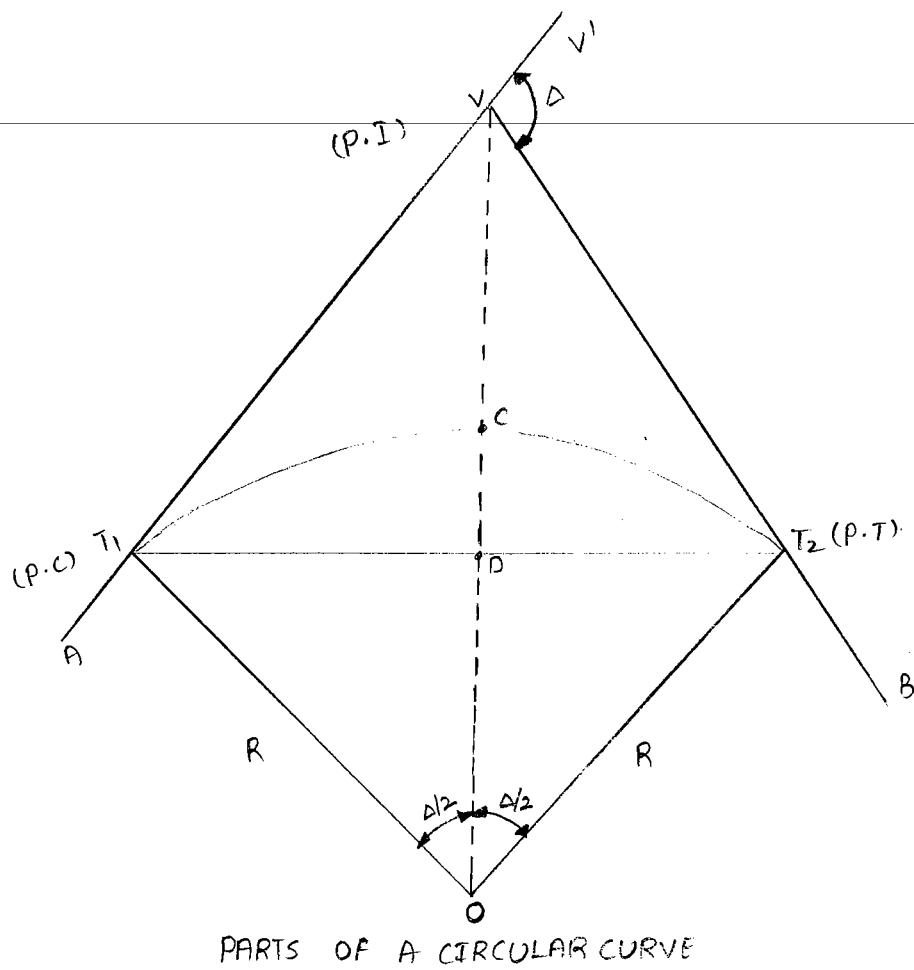
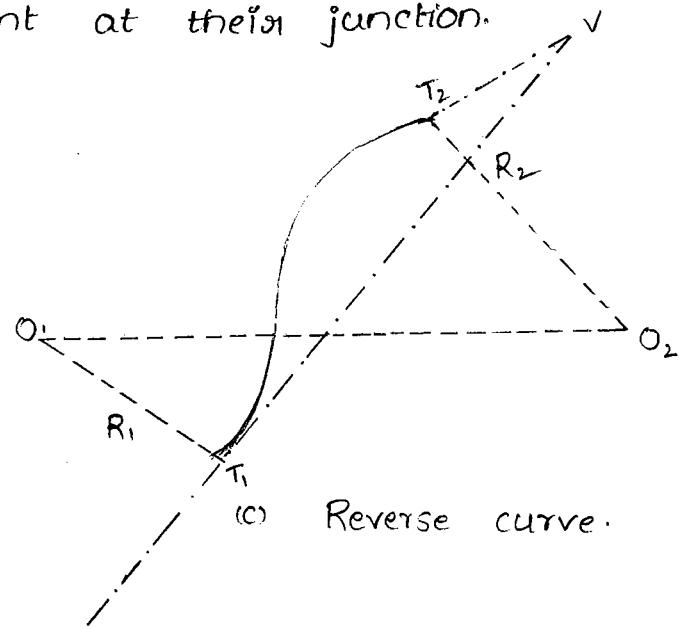
* Simple curve :- A curve is the one which consists of a single arc of a circle. it is tangential to both the straight line.



* Compound curve :- A compound curve consists of two or more simple curves that turn in the same direction and join at common tangent points.



* Reverse curve :- A reverse curve is the one which consists of two circular arcs of same or different radii, having their centres to the different sides of the common tangent. Both the arcs thus bend in different directions with a common tangent at their junction.



PARTS OF A CIRCULAR CURVE

- * **Back tangent** :- The tangent (T_1) previous to the curve is called the back tangent (or) first tangent.
- * **Forward tangent** :- The tangent (T_2) following the curve is called the forward tangent (or) Second tangent.
- * **point of intersection** :- If the two tangents AT_1 and BT_2 are produced, they will meet in a point, called the point of intersection (P.I) or vertex (V).
- * **point of curve** :- It is the beginning of the curve where the alignment changes from a tangent to a curve.
- * **point of tangency** :- It is end of the curve where the alignment changes from a curve to tangent.
- * **Intersection angle** :- The angle $\angle VVB_1$ between the tangent AV produced and VB is called the intersection angle (A) or the external deflection angle between the two tangents.
- * **Deflection angle to any point** :- The deflection angle to any point on the curve is the angle at p.c between the back tangent and the chord from p.c to point on the curve.
- * **Tangent distance** :- It is the distance between p.c to P.I [also the distance from P.I to P.T].
- * **External distance** :- (E) : It is the distance from the mid-point of the curve to P.I
- * **length of curve** :- (L) It is the total length of the curve from p.c to P.T.
- * **Long chord** :- It is chord joining p.c to P.T.
- * **Mid - Ordinate** :- (M) : It is the ordinate from the mid-point of the long chord to the mid-point of the curve.
- * **Normal chord** :- A chord between two successive regular stations on a curve.
- * **Sub-chord** :- Sub-chord is any chord shorter than the normal chord.

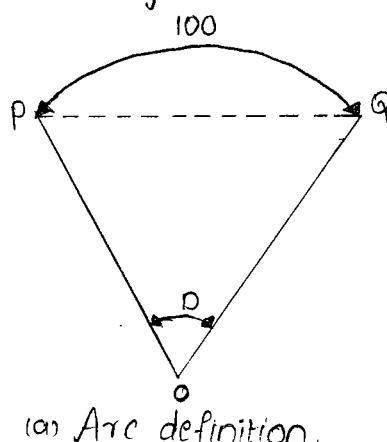
- * Right-hand curve:- If the chord curve deflects to the right of the direction of the progress of Survey, it is called the right-hand curve.
- * Left-hand curve:- If the curve deflects to the left of the ~~distance~~ direction of the progress of Survey, it is called the left-hand curve.

Designation of curve:-

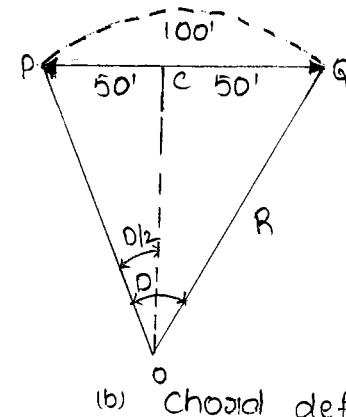
The Sharpness of the curve is designated either by its radius or by its "degree of curvature".

According to the arc definition, generally used in highway practice, The degree of the curve is defined as the central angle of the curve that is Subtended by an arc of 100 ft length.

According to the chord definition, generally used in rail-way practice, the degree of the curve is defined as the central angle of the curve that is subtended by its chord of 100 ft length.



(a) Arc definition.



(b) Chord definition.

Arc definition :- we have,

$$100 : 2\pi R = D : 360^\circ$$

$$R = \frac{360^\circ}{D} \times \frac{100}{2\pi} = \frac{5729.578}{D} \text{ ft.}$$

Thus, radius of 1° curve is 5729.578 ft. To the first approximation, we have

$$R = \frac{5730}{D}$$

* Chord definition:- From 1st POQ [Fig. b]

$$\sin \frac{1}{2}D = \frac{50}{R}$$

$$R = \frac{50}{\sin \frac{1}{2}D} \text{ (exact).}$$

When D is small, $\sin \frac{1}{2}D$ may be taken approximately equal to $\frac{1}{2}D$ radians.

$$R = \frac{50}{\frac{D}{2} \times \frac{\pi}{180}}, \text{ where } D \text{ is the degrees.}$$

$$= \frac{50 \times 360}{D \times \pi} = \frac{5729.578}{D} = \frac{5730}{D} \text{ (approximately).}$$

* Setting out Simple curves:-

The methods of setting out curves can be mainly divided into two heads depending upon instruments used:

i) Linear methods:- In the linear methods, only a chain or tape is used. Linear methods are used when

- A high degree of accuracy is not required.
- The curve is short.

ii) Angular methods:- In the angular method, an instrument such as a theodolite is used with tape, without a chain or tape. Before a curve is set out, it is essential to locate the tangents point of intersection (P.I), point of curve (P.C) and point of tangency (P.T).

* Linear methods of setting out :-

Following are some of the linear methods for setting out simple circular curves:

1. By ordinates or offsets from the long chord.
2. By successive bisection of arcs.
3. By offsets from the tangents.
4. By offsets from chords produced or by deflection distances.

* Compound curves:-

- * Setting out compound curve:- The compound curve can be set by method of deflection angles. The first branch is set out by setting the theodolite at T₁ (P.C) and the second branch is set out by setting the theodolite at the point D (P.C.C). The procedure is as follows:
- 1, After having known any four parts, calculate the rest of the three parts by the formulae developed in § 2.2.
 - 2, knowing T_s and T_b, Locate points T₁ and T₂ by linear measurements from point of intersection.
 - 3) Calculate the length of curves l_s and l_b. calculate the chainage of T₁D and T₂ as usual.
 - 4, For the first curve, calculate the tangential angles etc., for setting out the curve by Rankine's method.
 - 5) Set the theodolite at T₁ and set out the first branch of the curve as already explained.
 - 6, After having located the last point D (p.c.c) Shift the theodolite to D and set it there. with the vernier set to $(360^\circ - \frac{\Delta_1}{2})$ reading, take a backsight on T₁ on plunge the telescope. The line of sight is thus oriented along T₁D produced and if the theodolite is now swung through $\frac{\Delta_1}{2}$, the line of sight will be directed along the common tangent DD₂. Thus the theodolite is correctly oriented at D.
 - 7, Calculate the tangential angles of the second branch and set out the curve by observations from D, till T₂ is reached.
 - 8, Check the observations by measuring the angle T₁DT₂, which should be equal to $(180^\circ - \frac{\Delta_1 + \Delta_2}{2})$ or $(180^\circ - \frac{\Delta}{2})$.

* Geodetic Surveying :-

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- * The object of the geodetic surveying is to determine very precisely the relative or absolute positions on the earth's surface of a system of widely separated points.
- * The relative positions are determined in terms of the lengths and azimuths of the lines joining them.
- * The absolute positions are determined in terms of latitude, longitude and elevation above mean sea level.
- * The distinction between geodetic surveying and plane surveying is fundamentally one of extent of area rather than of operations.
- * The precise methods of geodesy are followed in the field work of extensive plane trigonometrical surveys also.
- * Since the area embraced by a geodetic survey from an appreciable portion of the surface of the earth, the sphericity of the earth is taken into consideration while making the computation.
- * Geodetic work is usually undertaken by the State Agency.

* Total Station :-

- * A total station is a combination of an electronic theodolite and an electronic distance meter (EDM).
- * This combination makes it possible to determine the co-ordinates of a reflector by aligning the instruments cross-hairs on the reflector and simultaneously measuring the vertical and horizontal angles and slope distances.
- * A micro - processor in the instrument takes care of recording reading and the necessary computations.
- * The data is easily transferred to a computer where it can be used to generate a map.
- * Wild Tachymat TC 2000, described in the previous article is one such total station manufactured by M/s "Wild Heerbrugg".

* In the field, it requires team work, planning, and careful observations.

* The more the user understands how a total station works, the better they will be able to use it.

Fundamental measurements: When aimed at an appropriate target

a total station measures three parameters. See the Fig.a

1. The rotation of the instrument's optical axis from the instrument north in a horizontal plane : i.e., Horizontal angle.
2. The inclination of the optical axis from the local vertical i.e., Vertical angle.

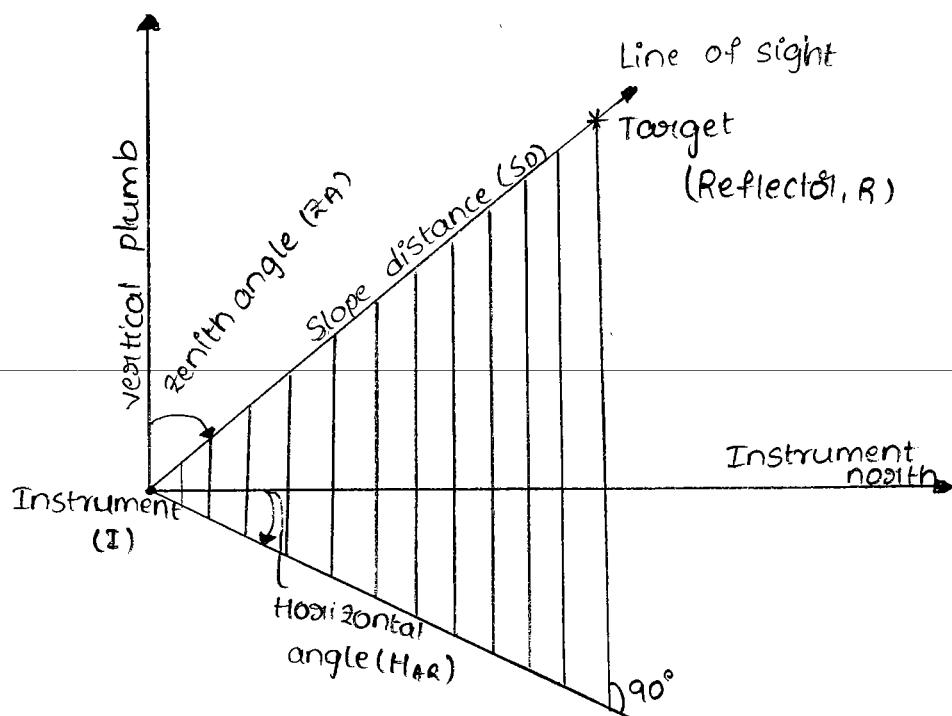


Fig.(a)

Fundamental measurements made by a Total station.

3. The distance between the instrument and the target .
i.e., Slope distance.

All the numbers that may be provided by the total station are derived from these fundamental measurements .

* Global positioning System :- (GPS)

* SR9400 GPS Sensor:-

New possibilities for single-frequency GPS:

The high-accuracy carrier phase and ultra-precise code measurements, the SR9400 opens up many new possibilities for single-frequency surveying:

1. Control, detail, topographic and engineering surveys to centimeter accuracies with differential phase.
2. GIS, mapping, seismic and hydrographic surveys with sub-half-meter positioning with differential code.
3. Data recording and post processing.
4. Real-time GPS surveying.

* Cost Effective to GPS cost-effective Solutions

GPS has attractive price and versatility, the SR9400 provides easy entry to GPS Surveying and cost effective solutions for many tasks:

- * Control Surveys with short and medium lines, when very short observation times are not essential and the influence of the ionosphere is relatively small.
- * Kinematic Surveys.
- * Real-time Surveys.

* SR9400 GPS Sensor with a TAOI Antenna:-

1. code (pseudorange) measurements of remarkably high precision for 30-50 cm differential positioning.
2. High-accuracy, carrier-phase measurements for centimeter level work.
3. Highest-possible signal strengths for reliable satellite tracking to low elevations and under poor conditions.

For DGPS applications, RTCM V2.0 output and input are available via the CR344 controller, reference-station software and SPCs Software; NMEA sentences can be output.

* Functioning of GPS controllers:-

The SR9400 connects to the CR333 and CR344 controllers. All the features, functions and operating comfort of System 300 are provided.

* Real-time GPS Surveying:-

When connected to a CR344 controller and radio modem, the SR9400 can be used very effectively for real-time surveying and setting out. Depending on the mode, achievable in real time are:

* 10-20mm +2ppm with differential phase.

* 30-50cm with differential code.

* By ordinates from the long chord :-

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Let R = Radius of the curve

O_o = Mid-ordinate

O_x = Ordinate at distance ' x ' from
the mid-point of the chord.

T_1 and T_2 = Tangent points.

L = Length of the long chord
actually measured on the
ground.

Bisect the long chord at point D .

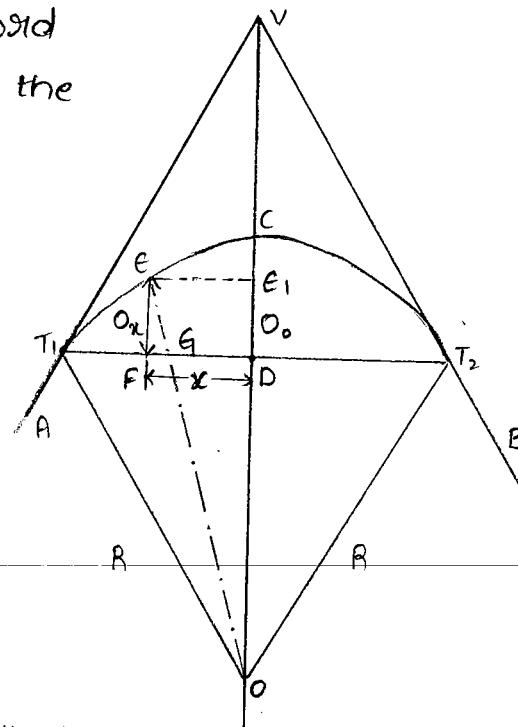
from $\Delta O T_1 D$

$$O T_1^2 = T_1 D^2 + O D^2$$

$$R^2 = \left(\frac{L}{2}\right)^2 + (CO - CD)^2 = \left(\frac{L}{2}\right)^2 + (R - O_o)^2.$$

$$(R - O_o) = \sqrt{R^2 - \left(\frac{L}{2}\right)^2} \quad (\text{or})$$

$$O_o = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}.$$



By ordinates from the long chord

In order to calculate the ordinate O_x to any point E , draw the line EE_1 parallel to the long chord $T_1 T_2$. join EO to cut the long chord in G .

$$\text{Then } O_x = EF = E_1 D$$

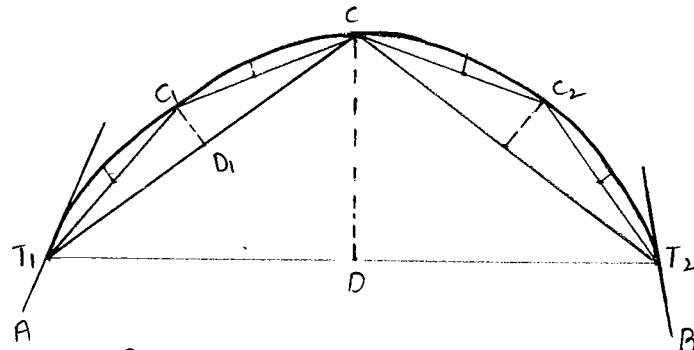
$$= E_1 D - O D$$

$$= \sqrt{(EO)^2 - (E E_1)^2} - (CO - CD)$$

$$= \sqrt{R^2 - x^2} - (R - O_o) \rightarrow \text{Exact.}$$

The long chord is divided into an even number of equal parts. offsets calculated from this equation.

* By Successive Bisection of Arcs of chords :-



Successive Bisection of arcs

* Join the tangent point T_1, T_2 and bisect the long chord at D .

Erect the perpendicular DC and make it equal to the versed sine of the curve. Thus,

$$\begin{aligned} CD &= R \left(1 - \cos \frac{\Delta}{4}\right) \\ &= R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}. \end{aligned}$$

* Join T_1C and T_2C and bisection them at D_1 and D_2 respectively. At D_1 and D_2 , Set out perpendicular offsets $c_1D_1 = c_2D_2 = R \left(1 - \cos \frac{\Delta}{4}\right)$, to get points C_1 and C_2 on the curve.

* By the successive bisection of these chords, more points may be obtained.

* By Offsets from the tangents :- The offsets from the tangents can be of two types:- i) Radial offsets ii) perpendicular offsets.

* Radial offsets :-

let O_x = Radial offset DE at any distance 'x' along the tangent.

$$T_1D = x.$$

from $\Delta^{1e} T_1DO$,

$$DO^2 = T_1O^2 + T_1D^2$$

$$(DE + EO)^2 = T_1O^2 + T_1D^2.$$

$$(DE)^2 = (O_x + R)^2 = R^2 + x^2$$

$$O_x = \sqrt{R^2 + x^2} - R. \text{ for exact.}$$

approximate expression for O_x

expand $\sqrt{R^2+x^2}$, Thus,

$$O_x = R \left(1 + \frac{x^2}{2R^2} - \frac{x^4}{8R^4} + \dots \right) - R.$$

Now, neglect the other two except the first two, we get,

$$O_x = R + \frac{x^2}{2R} - R$$

$$O_x = \frac{x^2}{2R} \dots \text{(approximate).}$$

then,

$$T_1 D^2 = DE (2R + DE)$$

$$x^2 = O_x (2R + O_x)$$

Neglecting O_x in comparison to $2R$, we get

$$O_x = \frac{x^2}{2R} \text{ (app).}$$

ii) perpendicular offsets:

Setting out by Radial offsets.

let $DE = O_x = \text{offset perpendicular to the tangent.}$

$T_1 D = x$; measured along the tangent

Draw EE_1 parallel to the tangent.

As EE_1O , we have.

$$E_1 O^2 = E_1 E^2.$$

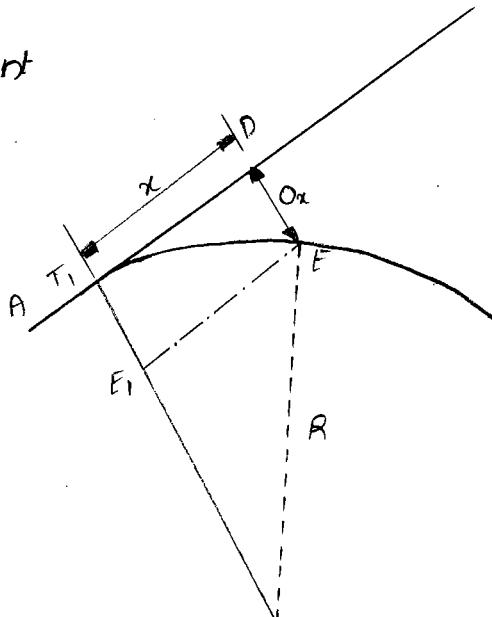
$$(T_1 O - T_1 E_1)^2 = E_1 O^2 - E_1 E^2.$$

$$(R - O_x)^2 = R^2 - x^2.$$

From which, $O_x = R - \sqrt{R^2 - x^2}$ (exact).

approximate expression for O_x .

expanding the term $\sqrt{R^2 - x^2}$, Thus,



Setting out by perpendicular offsets.

$$O_x = R - R \left(1 - \frac{x^2}{2R^2} - \frac{x^4}{8R^4} \dots \right)$$

$$O_x = R - R + \frac{x^2}{2R}$$

$$\boxed{O_x = \frac{x^2}{2R}}$$

* By deflection distances (offsets) offsets from the chords produced:-

The method is very much useful for long curves and is generally used on highway curves when a theodolite is not available.

Let $T_1 A_1 = T_1 A = \text{initial sub-chord}$
 $= C_1$

A_1, B_1, C etc... = points on the curve

$$AB = C_2$$

$$BD = C_3 \text{ etc...}$$

$T_1 V$ = Rear Tangent

$\angle A_1 T_1 A = S$ = deflection angle of the first chord.

$A_1 A = O_1$ = first offset.

$B_2 B = O_2$ = Second offset.

$D_3 D = O_3$ = Third offset, etc.

Now arc $A_1 A = O_1 = T_1 A \cdot S \rightarrow i$,

since $T_1 V$ is the tangent to the circle at T_1 ,

$$\angle T_1 O A = \angle A_1 T_1 A = 2S$$

$$T_1 A = R \cdot 2S$$

$$S = \frac{T_1 A}{2R}$$

Sub in eq ① value of S , we get,

$$\text{Arc } A_1 A = O_1 = T_1 A \cdot \frac{T_1 A}{2R} = \frac{T_1 A^2}{2R}$$

Taking arc $T_1 A = \text{chord } T_1 A$, we get,

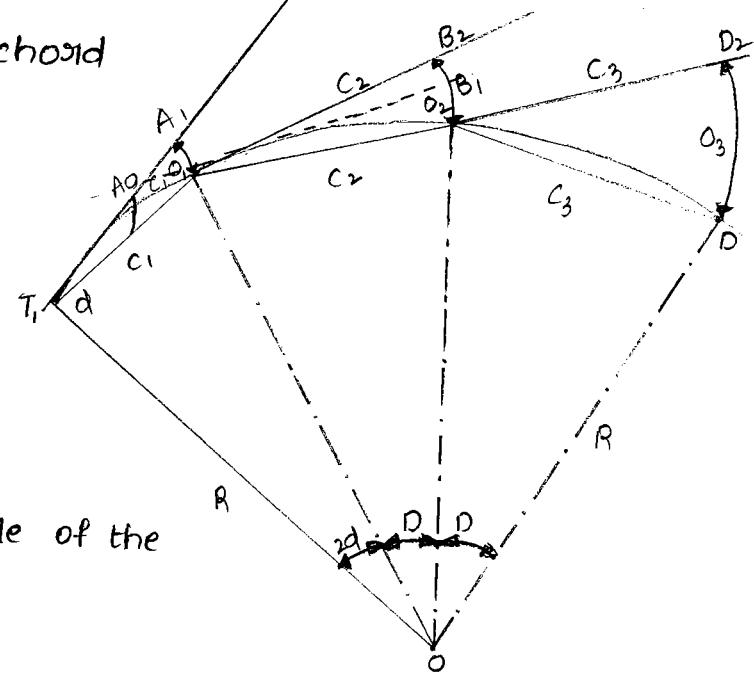
$$O_1 = \frac{C_1^2}{2R}$$

The offset $B_2 B$ from the tangent $A_1 B_1$ is given by

$$B_2 B = \frac{C_2^2}{2R}$$

Again, $\angle B_2 A_1 B_1 = \angle A_1 T_1 A$, being opposite angles.

Since, $T_1 A'$ and $A_1 A$ are both tangents, they are equal in length



$$\underline{[A'AT]}_1 A = S = \underline{[A'AT]}_1$$

$$\underline{[B_2AB_1]} = \underline{[A'AT]}_1 = S$$

$$\text{arc } B_2 B_1 = AB_2 \cdot \delta = C_2 \cdot \delta$$

Sub the value of S from (ii), we get,

$$B_2 B_1 = C_2 \cdot \frac{T_1 A}{2R} = \frac{C_2 \cdot C_1}{2R}$$

$$\text{arc } B_2 B = B_2 B_1 + B_1 B.$$

$$\Rightarrow O_2 = \frac{C_2 C_1}{2R} + \frac{C_2^2}{2R} = \frac{C_2}{2R} (C_1 + C_2)$$

Similarly, The third offset $O_3 = D_2 D$ is given by

$$O_3 = \frac{C_3}{2R} (C_2 + C_3)$$

Last or n th offset is given by

$$O_n = \frac{C_n}{2R} (C_{n-1} + C_n).$$

Generally, the first chord is a sub-chord length c , and the intermediate chords are normal chords length c . In that case, the above formulae reduce to

$$O_1 = \frac{c^2}{R}$$

$$O_2 = \frac{c}{2R} (c + C)$$

$$O_3 = O_4 = \dots O_{n-1} = \frac{c}{2R} (2c) = \frac{c^2}{R}$$

$$O_n = \frac{c'}{2R} (c + c')$$

where ' c' ' is the last sub-chord.

"Instrumental methods":-

The following are instrumental methods commonly used for setting out a circular curve:

- (1) Rankine's method of tangential (or) deflection angle.
- (2) Two theodolite method.
- (3) Tacheometric method.

* Rankine's method:- A Deflection angle to any point on the curve is the angle at P_c between the back tangent and the chord from the P_c to that point.

The Rankine's method is based on the principle that the deflection angle to any point on the circular curve is measured by half of the angle subtended by the arc from point of curvature to that point.

It is assumed that the length of arc is approximately equal to its curve.

T_1N = A Radio tangent.

T_1 = point of curvature of given circular curve.

S_1, S_2, S_3 = The tangential angle ($^{\circ}$)

the angles which are present successive chord T_1A, AB & BC etc....

These are makes angle with the representative tangent to the curve at T_1, A, B .

$\Delta_1, \Delta_2, \Delta_3$ = These are the total tangential angles ($^{\circ}$) deflection angles to the points A, B, C .

$\therefore c_1, c_2, c_3$ are the length of the chord.

$$T_1A = c_1$$

$$AB = c_2$$

$BC = c_3$ respectively.

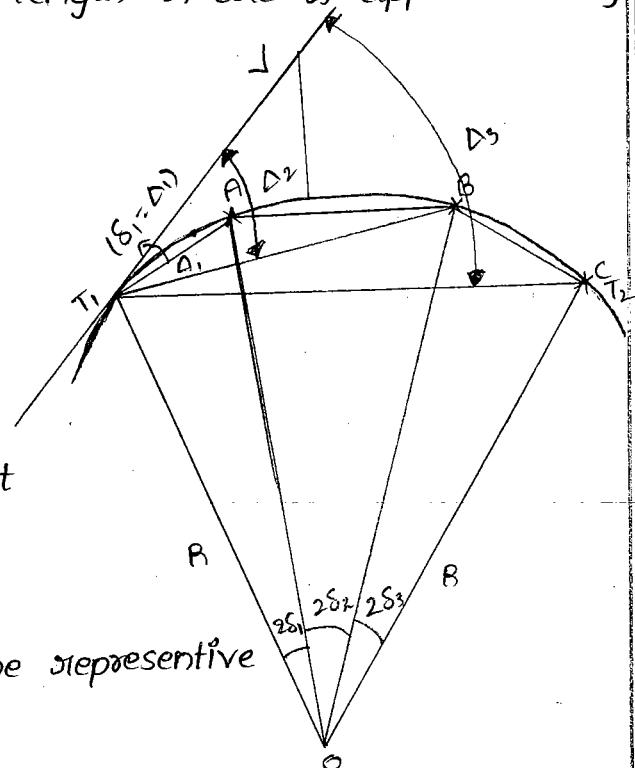
Finally, calculated deflection angle,

$$S_1 = 1718 \cdot a \frac{c_1}{R} \text{ min}$$

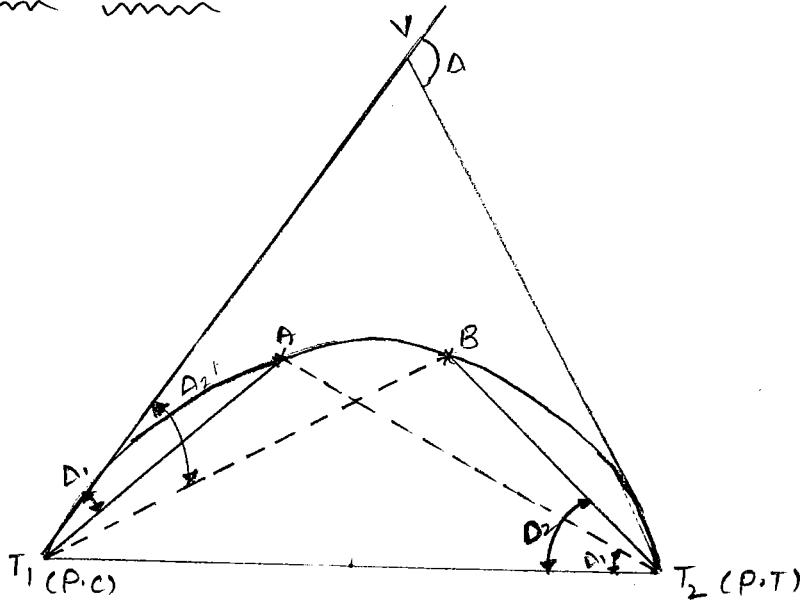
$$S_1 = \Delta_1$$

$$\Delta_2 = S_1 + S_2$$

$$\Delta_3 = S_1 + S_2 + S_3$$



Two Theodolite method :-



Two - Theodolite method.

* In this method two theodolites are used one at point of curvature T_1 and another at P_T i.e., at T_2 , the method is used when the ground is unsuitable for chaining and it is based on the principle of the angle between Tangent and the chord is equal to the angle which is the chords subtended in the opposite Segment.

$$\underline{VT_1A} = \Delta,$$

i.e., Deflection angle for A . but $\underline{AT_2T_1}$ is the angle subtended by the chord T_1A in the opposite Segment

$$\therefore \underline{AT_2T_1} = \underline{VT_1A} = \Delta,$$

hence $\underline{BT_1V} = \underline{TT_2B} = \Delta$, Hence the angle between the long chord and the line joining any point to T_2 is equal to the deflection angle to the point measured with the rare tangent i.e., "A".

* Tacheometric Method :-

Formulae :-

$$L = \frac{f}{i} s + (f+d), \text{ when the line of sight is horizontal}$$

$$L = \frac{f}{i} s \cos^2 \theta + (f+d) \cos \theta, \text{ when the line of sight is inclined.}$$

(Refer in text-book) (for more information).

Problems:-

1. calculate the coordinates at 10mts distances for a circular curve having a long chord of 80mts and a versed sine of 4mts.

Sol:- The versed sine is given by,

$$O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}.$$

$$4 = R - \sqrt{R^2 - (40)^2}$$

$$R^2 - (40)^2 = (R-4)^2 = R^2 + 16 - 8R.$$

$$R = \frac{1616}{8} = 202 \text{ metres.}$$

$$(R - O_0) = 202 - 4 = 198 \text{ m.}$$

we have,

$$O_{10} = \sqrt{R^2 - x^2} - (R - O_0).$$

$$O_{10} = \sqrt{(202)^2 - (10)^2} - 198 = 201.75 - 198 = 3.75 \text{ m}$$

$$O_{20} = \sqrt{(202)^2 - (20)^2} - 198 = 201.01 - 198 = 3.01 \text{ m.}$$

$$O_{30} = \sqrt{(202)^2 - (30)^2} - 198 = 199.76 - 198 = 1.76 \text{ m}$$

$$O_{40} = \sqrt{(202)^2 - (40)^2} - 198 = 198 - 198 = 0.$$

2. Determine the offsets to be set out at $\frac{1}{8}$ chain interval along the tangents to locate a 16-chain curve, the length of each chain being 20 m.

we have,

$$O_x = \sqrt{R^2 + x^2} - R \quad \text{Here } R = 16 \text{ chains.}$$

$$O_{0.5} = \sqrt{(16)^2 + (0.5)^2} - 16 = 0.0048 \times 20 \text{ chains} = 0.15625 \text{ mts}$$

$$O_1 = \sqrt{(16)^2 + (1)^2} - 16 = 0.081 \times 20 \text{ chains} = 0.62 \text{ m.}$$

$$O_{1.5} = \sqrt{(16)^2 + (1.5)^2} - 16 = 0.0702 \times 20 \text{ chains} = 1.40 \text{ mts.}$$

$$O_2 = \sqrt{(16)^2 + (2)^2} - 16 = 0.1245 \times 20 \text{ chains} = 2.49 \text{ mts.}$$

$$O_{2.5} = \sqrt{(16)^2 + (2.5)^2} - 16 = 0.1941 \times 20 \text{ chains} = 3.88 \text{ mts.}$$

$$O_3 = \sqrt{(16)^2 + (3)^2} - 16 = 0.2788 \times 20 \text{ chains} = 5.58 \text{ mts.}$$

COMPUTATION OF AREAS & VOLUMES

Introduction for Computation of Areas & volumes :-

Computation of Area :-

- The area is required for the title documents of the land.
- The catchment area of a river is required for the design of bridges, dams, reservoirs etc.
- When the plan is enclosed by straight boundaries, it can be subdivided into simple geometrical shapes, such as triangles, trapezoids, rectangles etc.

Note :- 1 hectare = 10^4 m^2 .

1 Sq. Kilometer = $10^6 \text{ m}^2 = 100$ hectares.

Computation of volume :-

- The volume of earthwork is required for the selection of a suitable alignment for a road, canal or sewer.
- The computation of the volume of earthwork is generally done after computing the areas of variouscls.
- For the estimation of the volume of water in a reservoir, the contour map is generally used.

Area along irregular boundaries :-

If the boundaries of a tract are irregular, it is not possible to run the traverse along the boundaries. The area between the traverse line and the irregular boundary is determined using the following methods :-

1. Mid - ordinate rule :-

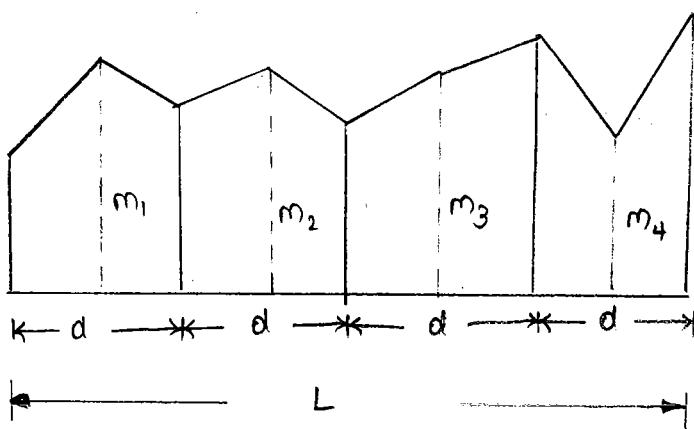
In this method, the tract is divided into segments, and lengths of middle ordinates of each segment is measured.

$$A = d [o_1 + o_2 + o_3 + \dots]$$

$$A = \frac{L}{n} [o_1 + o_2 + o_3 + \dots]$$

where n = no. of segments

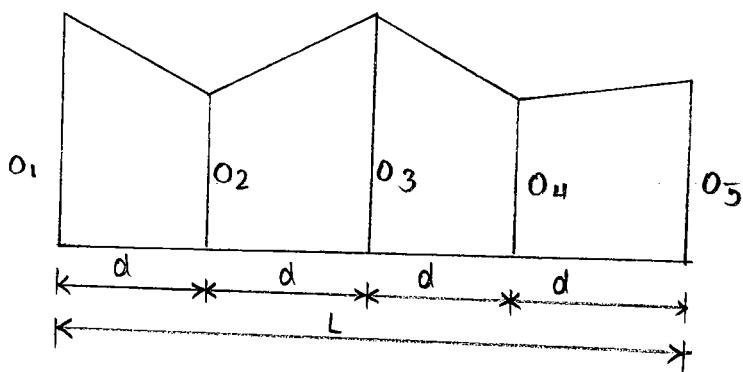
L = length of the base line.



Average- Ordinate method :-

In this method , the length of average ordinate is determined and is used to determine the area of the entire tract .

$$\text{Average Ordinate} = \frac{O_1 + O_2 + O_3 + O_4 + O_5}{5}$$



$$\text{Area} = \text{Av. Ordinate} \times L.$$

$$A = \frac{[O_1 + O_2 + \dots + O_{n+1}]}{n+1} \times L.$$

* 3. Trapezoidal rule :-

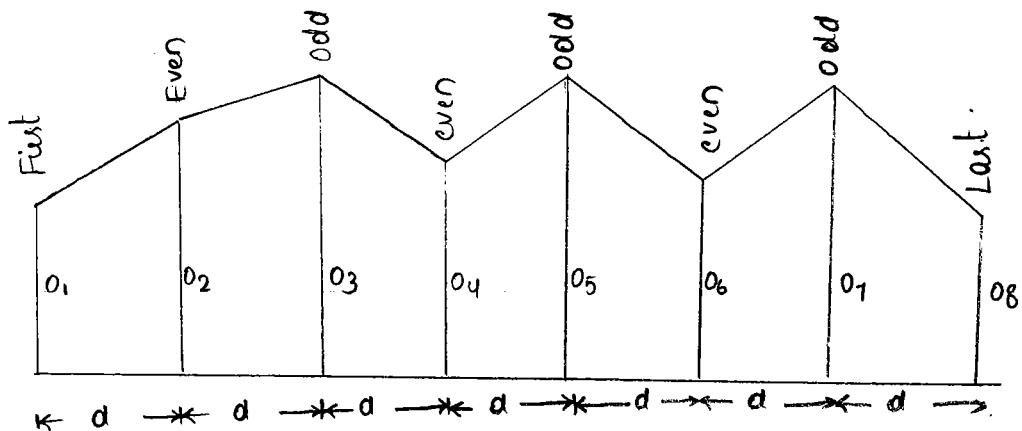
In this method , the tract is divided into a no. of trapezoids, and area is determined Separately .

$$A = \frac{L}{n} \left[\frac{O_1 + O_{n+1}}{2} + O_2 + O_3 + \dots + O_n \right]$$

$$A = L \left[\frac{O_1 + O_{n+1}}{2} + O_2 + O_3 + \dots + O_n \right].$$

Simpson's Rule :-

$$A = \frac{d}{3} \left[(O_1 + O_{n+1}) + 4(O_2 + O_4 + O_6 + \dots + O_n) + 2(O_3 + O_5 + O_7 + \dots + O_{n-1}) \right]$$



Problems on the trapezoidal & Simpson's rule:-

1. The following læ offsets were taken at 5m intervals from a traverse line to an irregular boundary line. 2.10, 3.15, 4.50, 3.60, 4.58, 7.85, 6.45, 4.65, 3.14 m.

(a) Simpson's rule (b) Trapezoidal rule

(a) Simpson's rule :-

$$d = 5\text{m}, O_1 = 2.10\text{m}, O_2 = 3.15\text{m}, O_3 = 4.50\text{m}, O_4 = 3.60\text{m}$$

$$O_5 = 4.58\text{m}, O_6 = 7.85\text{m}, O_7 = 6.45\text{m}, O_8 = 4.65\text{m}, O_9 = 3.14\text{m}.$$

SOL

$$\begin{aligned}
 A &= \frac{d}{3} \left[(O_1 + O_9) + 4(3.15 + 3.60 + 7.85 + 4.65) + 2(4.50 + 4.58 + 6.45) \right] \\
 &= \frac{5}{3} \left[(2.10 + 3.14) + 4(3.15 + 3.60 + 7.85 + 4.65) + 2(4.50 + 4.58 + 6.45) \right] \\
 &= 188.83 \text{ m}^2
 \end{aligned}$$

Trapezoidal rule :-

$$\begin{aligned}
 A &= \frac{d}{a} \left[(O_1 + O_9) + 2(O_2 + O_3 + O_4 + O_5 + O_6 + O_7 + O_8) \right] \\
 &= \frac{5}{2} \left[(2.10 + 3.14) + 2(3.15 + 3.60 + 7.85 + 4.65 + 4.50 + 4.58 + 6.45) \right] \\
 &= 187 \text{ mm}^2
 \end{aligned}$$

2. A series of offsets was taken from a base line to a curved boundary line at interval of 10m in the following order.
 $O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8$.

Ans

$$\begin{aligned}
 O_1 &= 0 \text{ m}, O_2 = 2.68 \text{ m}, O_3 = 3.64 \text{ m}, O_4 = 3.70 \text{ m}, O_5 = 4.60 \text{ m} \\
 O_6 &= 3.62 \text{ m}, O_7 = 4.84 \text{ m}, O_8 = 5.74 \text{ m}.
 \end{aligned}$$

Trapezoidal rule :-

$$\begin{aligned}
 \text{Area} &= \frac{10}{3} \left[\frac{(0+5.74)}{2} + 2.68 + 3.64 + 3.70 + 4.60 + 3.62 + 4.84 \right] \\
 &= 10 \left[2.87 + 23.08 \right] \\
 &= 259.5 \text{ m}^2
 \end{aligned}$$

Simpson's rule :-

$$\text{Area} = \frac{10}{3} \left[(0+4 \cdot 84) + 4(2.68 + 3.70 + 3.62) + 2(3.64 + 4.60) \right]$$

$$= \frac{10}{3} [4 \cdot 84 + 40 + 16 \cdot 48]$$

$$A_1 = 204.4 \text{ m}^2$$

$$A_2 = \frac{10}{2} \left[4 \cdot 84 + 5 \cdot 74 \right]$$

$$= 52.9 \text{ m}^2$$

$$\text{Total Area} = A_1 + A_2 = 204.4 + 52.9 = 257.3 \text{ m}^2$$

3. The following offsets were taken from a chainline to an irregular boundary line at an interval of 10mts
 $O_1 = 0 \text{ m}$, $O_2 = 2.50 \text{ m}$, $O_3 = 3.50 \text{ m}$, $O_4 = 5 \text{ m}$, $O_5 = 4.60 \text{ m}$,
 $O_6 = 3.20 \text{ m}$, $O_7 = 0 \text{ m}$.

(a) Trapezoidal rule (b) Simpson's rule.

Sol

(a) Trapezoidal rule = $\frac{d}{2} \left[(O_1 + O_7) + 2(O_2 + O_3 + O_4 + O_5 + O_6) \right]$

$$= \frac{10}{2} \left[(0+0) + 2(2.50 + 3.50 + 5 + 4.60 + 3.20) \right]$$

$$= 188 \text{ m}^2$$

(b) Simpson's rule = $\frac{d}{3} \left[(O_1 + O_7) + 4(\text{even ordinate}) + 2(\text{odd ordinate}) \right]$

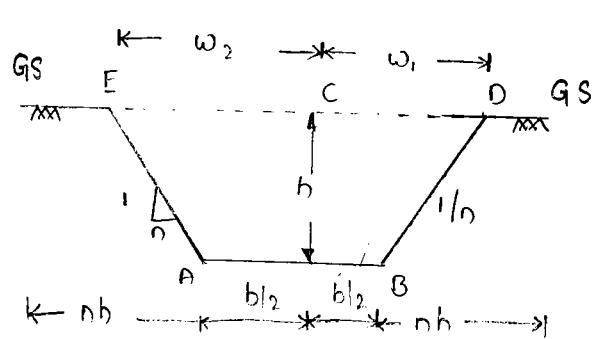
$$= \frac{10}{3} \left[(0+0) + 4(2.50 + 5 + 3.20) + 2(3.50 + 4.60) \right]$$

$$= 196.66 \text{ m}^2$$

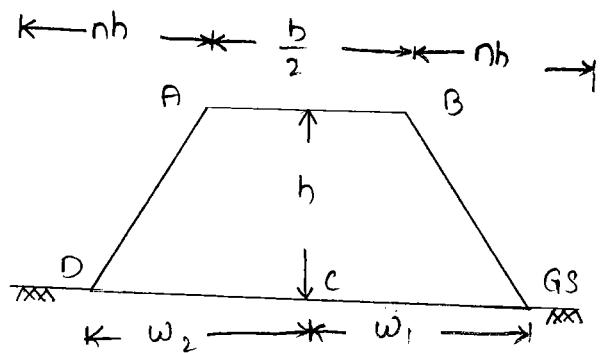
level - Section:-

Fig(a) shows a level section in cutting

Fig(b) shows a level section in filling .



(a)



(b)

Fig(a)(b) is the Constant formation width ,and h is the depth of cutting on the center line .The two Sides width w_1 and w_2 measured from the Center line to the point of intersection of the sides Slopes with the original ground Surface are Equal, i.e ,

$$w_1 = w_2 = w \text{ (Say)}$$

let the Side slopes be n to 1 , n horizontal to one Vertical

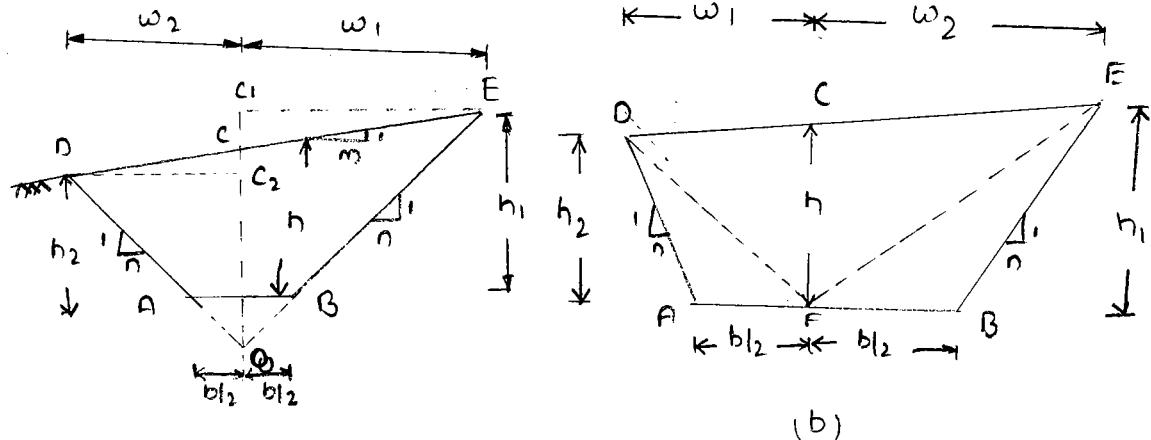
$$w = 0.5b + nh$$

$$2w = b + 2nh$$

$$A = \frac{(2w + b)}{2} \times h$$

$$= \frac{(b + 2nh + b)}{2} \times h \Rightarrow A = (b + nh)h.$$

Two-level Sections :-



(a)

If the transverse slope of the ground be m (horizontal) to (vertical). As the section is not symmetrical about the centre line, the side width w_1 and w_2 are not equal.

h = depth of cutting at the centre

h_1 and h_2 = depth of cutting at the two ends

h_1 and h_2 = difference b/w formation level AB

$$\text{Vertical intercept} = \frac{w_1}{m}$$

$$\text{Vertical intercept } (C_2 C) = \frac{w_2}{m}$$

$$\text{Vertical intercept } FO = \frac{b_1}{n} = \frac{b}{2n}$$

From Similar triangle GEO and FBO

$$\frac{C_1 E}{F B} = \frac{O C_1}{O F}$$

$$\frac{w_1}{b_1} = \frac{O F + F C + C_1 C}{O F}$$

$$= \frac{b/2n + h + w_1/m}{b/2n}$$

$$w_1 = \frac{w_1}{m} \times n + h \times n + \frac{b}{2}$$

$$w_1 \left(1 + \frac{n}{m}\right) = \left(\frac{b}{2} + nh\right)$$

$$w_1 = (b/2 + nh) \left(\frac{m}{mn}\right)$$

From Similar triangle C₂DO and FAO

$$\frac{DC_2}{AF} = \frac{C_2 O}{FO}$$

$$\frac{w_2}{b/2} = \frac{CF - CC_2 + FO}{FO}$$

$$= \frac{h - w_2/m + b/2n}{b/2n}$$

$$w_2 = nh - \frac{w_2}{m} \times n + \frac{b}{2}$$

$$w_2 \left(1 + \frac{n}{m}\right) = \frac{b}{2} + nh$$

$$w_2 = \left(\frac{b}{2} + nh\right) \left(\frac{m}{m+n}\right)$$

$$CO = h + FO$$

$$\text{height } CO = h + \frac{b}{2n}$$

Area ofcls DC EBA is given by

$$\text{Area } DC EBA = \text{Area } DCO + \text{Area } ECO - \text{Area } OAB$$

$$A = \left(\frac{1}{2} w_1\right) \times CO + \left(\frac{1}{2} w_2\right) CO - \frac{1}{2} b \times \frac{b}{2n}$$

$$A = \frac{1}{2} \omega_1 \left(h + \frac{b}{2n} \right) + \frac{1}{2} \omega_2 \left(h + \frac{b}{8n} \right) - \frac{b^2}{4n}$$

$$= \frac{1}{2} (\omega_1 + \omega_2) \left(h + \frac{b}{8n} \right) - \frac{b^2}{4n}$$

$$A = \frac{1}{2n} \left[\left(\frac{b}{2} + nh \right) (\omega_1 + \omega_2) \frac{b^2}{2} \right]. \quad \text{--- (1)}$$

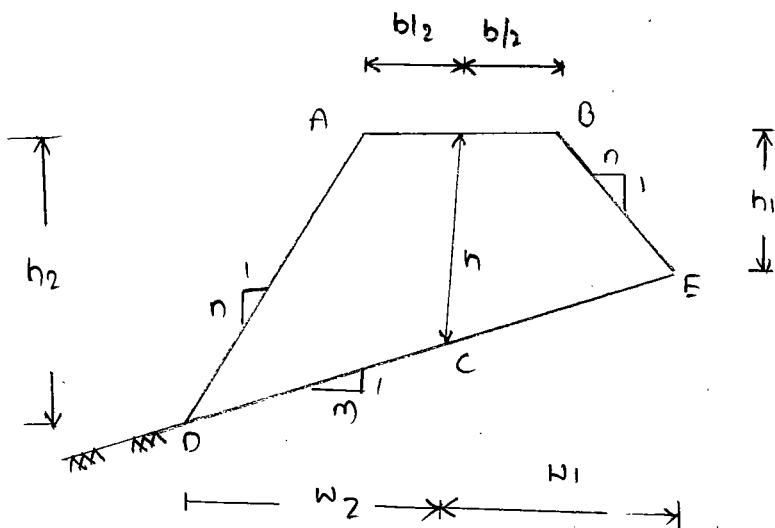
Eq. \rightarrow (1) can be modified by substituting the value ω_1 and ω_2 from eq.

$$\omega_1 + \omega_2 = \left(\frac{b}{2} + nh \right) \frac{m}{m+n} + \left(\frac{b}{2} + nh \right) \left(\frac{m}{m+n} \right)$$

$$A = \frac{1}{2n} \left[\left(\frac{b}{2} + nh \right) \left(\frac{b}{2} + nh \right) \frac{2m^2}{m^2-n^2} - \frac{b^2}{2} \right]$$

$$A = \frac{m^2 n}{m^2 - n^2} \left(h + \frac{b}{8n} \right)^2 - \frac{b^2}{2n}$$

Two-level Section in filling :-



In above figure, shows a two-level Section in filling. It can be shown that expressions for areas developed for the two-level section in cutting are also applicable to the two-level section in filling. However, in this case,

$$w_1 = \left(\frac{b}{2} + nh \right) \frac{m}{m+n}$$

$$w_2 = \left(\frac{b}{2} + nh \right) \left(\frac{m}{m-n} \right)$$

$$h_1 = h - \frac{w_1}{m} \quad \text{and} \quad h_2 = h + \frac{w_2}{m}.$$

Problem:-

1. A road embankment is 11m wide at formation level. The center line of the embankment is 3m above the ground surface. If the ground slope is 1 in 22 at right angles to the center line, and the side slopes are 2:1, calculate the side width and area ofcls by various formulae.

Sol Given data $\Rightarrow \frac{b}{2} = 5.5 \text{ m}$

$$m = 22, n = 2$$

$$h = 3$$

$$w_2 = \left(\frac{b}{2} + nh \right) \left(\frac{m}{m-n} \right)$$

$$= (5.5 + (2 \times 3)) \frac{22}{22-2} = 14.65 \text{ m}$$

$$w_1 = \left(\frac{b}{2} + nh \right) \left(\frac{m}{m+n} \right)$$

$$= (5.5 + (2 \times 3)) \left(\frac{22}{22+2} \right)$$

$$= 10.54 \text{ m}$$

$$A = \frac{1}{2n} \left[(b)_2 + nh \right] (w_1 + w_2) - \frac{b^2}{2}$$

$$= \frac{1}{2 \times 2} \left[(5.5 + (2 \times 3)) (12.65 + 10.54) - \frac{b^2}{2} \right]$$

$$= 51.55 \text{ m}^2$$

$$A = \frac{m^2 n}{m^2 - n^2} \left[h + \frac{b}{2n} \right]^2 - \frac{b^2}{4n}$$

$$= \frac{(22)^2 \times 2}{(22)^2 - (2)^2} \left[3 + \frac{5.5}{2} \right]^2 - \left[\frac{(11)^2}{4 \times 2} \right]$$

$$= 66.68 - 15.13$$

$$= 51.55 \text{ m}^2.$$

$$A = \frac{1}{2} \left[\frac{b}{2} (h_1 + h_2) + h (w_1 + w_2) \right]$$

$$h_1 = h - \frac{w_1}{m} = 3 - \frac{10.54}{22} = 0.521 \text{ m}$$

$$h_2 = h + \frac{w_2}{m} = 3 + \frac{12.65}{22} = 3.575 \text{ m}$$

$$A = \frac{1}{2} \left[5.5 (3.575 + 0.521) + 3 (12.65 + 10.54) \right]$$

$$= \frac{1}{2} [33.528 + 69.57] = 51.549 \text{ m} \text{ Say } 51.55 \text{ m}^2$$

$$\begin{aligned}
 A &= \frac{n\left(\frac{b}{2}\right)^2 + m^2(bh + nh^2)}{m^2 - n^2} \\
 &= \frac{2 \times (5.5)^2 + (22)^2 [11 \times 3 + 2 \times (3)^2]}{(22)^2 - (8)^2} \\
 &= \frac{60.5 + 484(33+18)}{484 - 4} \\
 &= 51.55 \text{ m}^2.
 \end{aligned}$$

Problem on Simpson's and Trapezoidal rule with unequal interval :-

9. The following offsets taken from a Survey line to a curved border line

Distance (D) 0 5 10 15 20 30 40 60 80

Offsets (in m) 2.50 3.80 4.60 5.20 6.10 4.70 5.80 3.90 2.20

$$d_1 = 5 \text{ m}, d_2 = 10 \text{ m}, d_3 = 20 \text{ m}$$

For (d_i)

$$\begin{aligned}
 (\text{a}) \text{ Trapezoidal rule} &= \frac{d}{2} [(o_1 + o_5) + 2(o_2 + o_3 + o_4)] \\
 &= \frac{5}{2} [(2.50 + 6.10) + 2(3.80 + 4.60 + 5.20)] \\
 &= 89.5 \text{ m}^2.
 \end{aligned}$$

Simpson's rule :-

$$A_1 = \frac{5}{3} [(2.50 + 6.20) + 4(3.80 + 5.20) + 2(4.60)] \\ = 89.83 \text{ m}^2.$$

For 'd₂'

$$A_2 = \frac{10}{2} [(6.10 + 5.80) + 2(4.70)] \rightarrow \text{Trapezoidal.} \\ = 106.5 \text{ m}^2$$

$$A_2 = \frac{10}{2} [(6.10 + 5.80) + 4(4.70)] \rightarrow \text{Simpson's} \\ = 102.33 \text{ m}^2.$$

For 'd₃'

$$A_3 = \frac{20}{2} [(5.80 + 2.20) + 2(3.90)] \rightarrow \text{Trapezoidal} \\ = 158 \text{ m}^2$$

$$A_3 = \frac{20}{3} [(5.80 + 2.20) + 4(3.90)] \rightarrow \text{Simpson's} \\ = 157.33$$

$$\text{Total area of Simpson's rule} = 157.33 + 102.33 + 89.83 \text{ m}^2 \\ = 349.49 \text{ m}^2.$$

$$\text{Total area of Trapezoidal rule } A = 158 + 106.5 + 89.5 \\ = 354 \text{ m}^2$$