

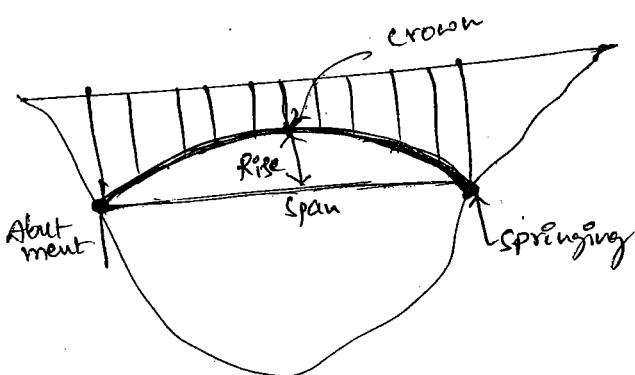
Three Hinged Arches: - Elastic theory of Arches - Eddy's theorem
 Determination of horizontal thrust, Bending moment,
 normal thrust and radial shear - effect of Temperature

Two Hinged Arches: - Determination of horizontal thrust, BM, normal
 thrust, radial shear - Rib shortening and temp stresses,
 tied arches - fixed arches (No analytical Qn)

Arches: - Intro:- For large spans, beams are very uneconomical and mostly the self wt of beams contributes to the stress in such large proportions that it is difficult to design beams for large spans.

e.g.: Bridges

Def: Arches are nothing but curved beams (vert plane) that transfer loads to their plane.



- Arches transfer loads to abutments at springing points.
- Hinges may be provided at these points.
- The top most point is crown
Some times has a hinge.
- The ht of crown above the support level is called rise.

- (Because of curved nature of arches, they develop horizontal forces)*
- Any section in the arch will be subjected to normal thrust, radial shear, and Bending moment. However BM is less compared to a beam of same span. Thus loads get transferred partly by axial compression and partly by flexure.

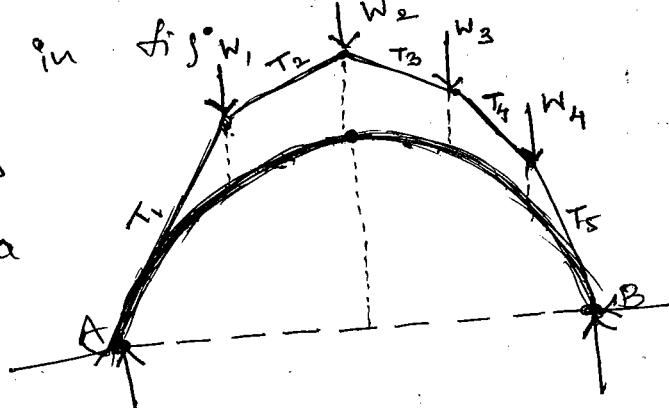
Internal forces in the Arch

Types of Arches (No. of Hinges)

- (i) Three - Hinged
- (ii) Two - Hinged
- (iii) Hinged & (fixed) (iv) Single Hinge

Elastic theory of Arches:- Consider an arch with

loads as shown in fig.



- Represent vertical loads to some scale on a vertical line.

- Select a point outside vertical line at some distance.

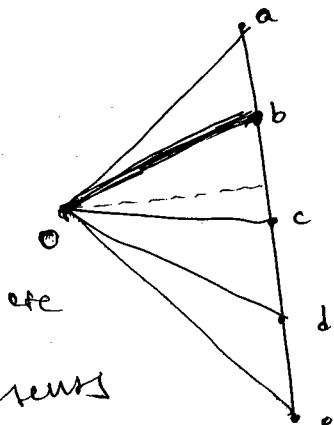
- Join Oa, Ob, Oc, Od etc

- Draw lines parallel to Oa, Ob etc on the arch, which represents T_1, T_2, \dots

- No. of polygons are possible as O can be chosen at any distance and anywhere.

→ Funicular polygon which lies within the actual arch is called as line of thrust.

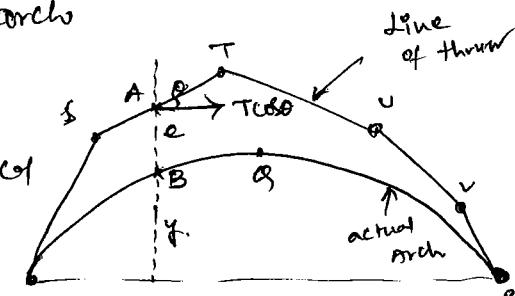
→ Polygonal arch following the path of true comp. is called theoretical Arch (S) Linear arch.



Eddy's theorem:- BM at any point on the arch axis is proportional to the vertical intercept b/w theoretical and actual arch.

proof :- Let PQR is actual arch

- Let PSTUVR is circular polygon representing theoretical arch.



- Select a point A' on line of thrust orough.
- draw a vertical line which will intersect actual arch at B.
- NCSD is the horz component of thrust N
- $BM = NCSD \times AB$.
- As. NCSD being constant throughout the arch,
- $BM \propto AB$

\therefore BM is proportional to vertical intercept.

Types of arches :- (Geometry)

- circular
- parabolic
- elliptical.

Circular Arch :-

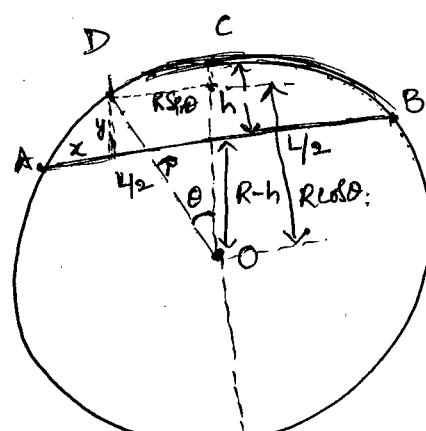
From the property of circle

$$\frac{L}{2} \times \frac{L}{2} = h(2R-h)$$

$$\frac{L^2}{4h} = 2R-h$$

$$2R = h + \frac{L^2}{4h}$$

$$R = \frac{h}{2} + \frac{L^2}{8h} \quad \leftarrow \text{radius of circular arch}$$



Taking Origin at A, $x = \frac{L}{2} - R \sin \theta$

$$y = R \cos \theta - (R - h)$$

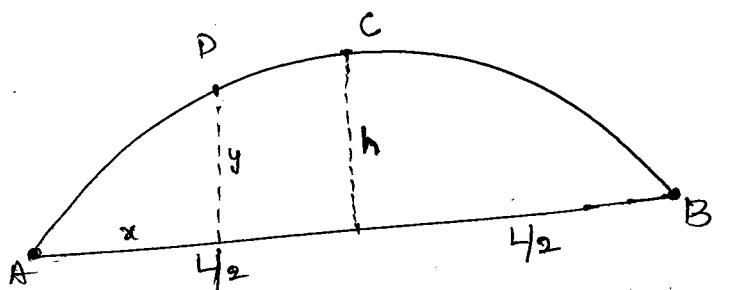
$$= R \cos \theta - R + h$$

$$= R (\cos \theta - 1) + h$$

$$\underline{y = h - R(1 - \cos \theta)}$$

For parabolic Arch Taking Origin at A:-

$$y = \frac{4hx}{L^2} (1-x)$$



if Origin is at 'C' :- $\frac{x^2}{y} = a$

$$\text{if } x = L/2, y = h \Rightarrow a = \frac{L^2}{4h}$$

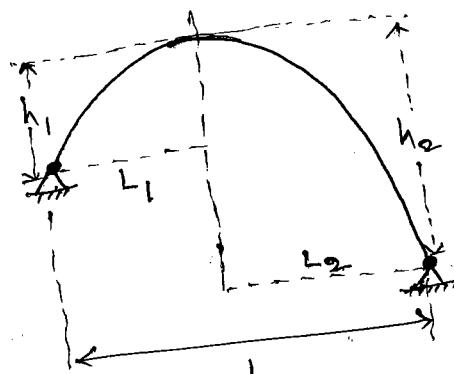
$$\frac{x^2}{y} = \frac{L^2}{4h} \Rightarrow y = \frac{4h x^2}{L^2}$$

if Springing points are not at same level:-

$$\frac{x^2}{y} = \text{constant}$$

$$\frac{x}{\sqrt{y}} = \text{constant}$$

$$\frac{L_1}{\sqrt{h_1}} = \text{const} : \frac{L_2}{\sqrt{h_2}} = \text{const}$$



$$a = \frac{L_1}{\sqrt{h_1}}, b = \frac{L_2}{\sqrt{h_2}} \Rightarrow a+b = \frac{L_1+L_2}{\sqrt{h_1}+\sqrt{h_2}}$$

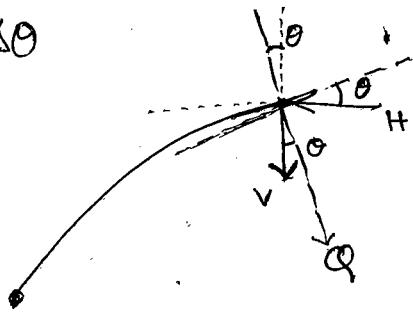
$$\frac{L_1}{a} = \frac{L_1+L_2}{\sqrt{h_1}+\sqrt{h_2}} \Rightarrow L_1 = \frac{L \sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} ; L_2 = \frac{L \sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$$

if a, b are constant
then L1 & L2 also constant

$$N \& Q := N = V \sin\theta + H \cos\theta$$

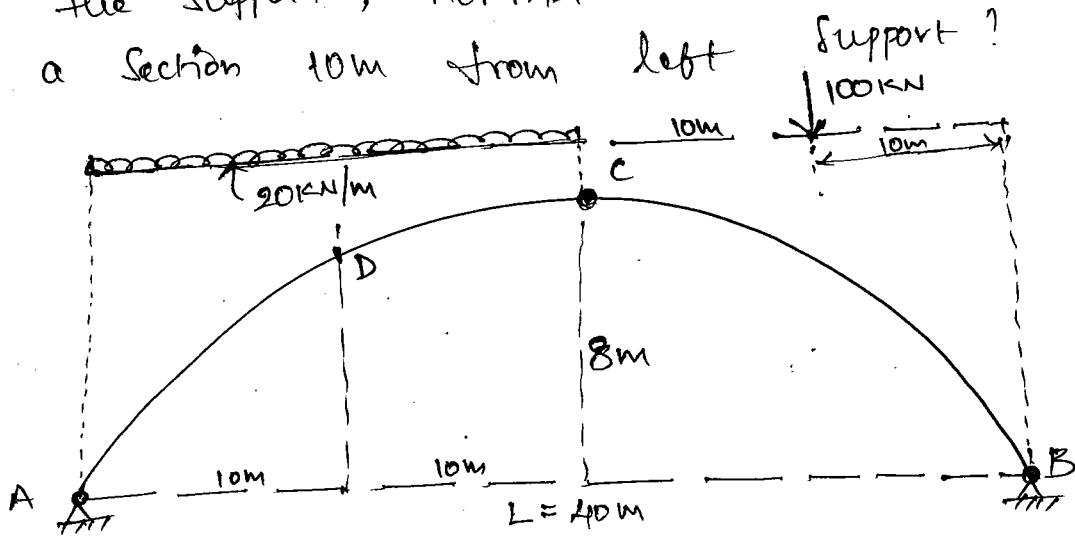
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$$Q = V \cos\theta - H \sin\theta$$



Prob 1:-
T50

Prob 1:- A 3H circular arch hinged at the springing and crown points has a span of 40m and a central rise of 8m. It carries a UDL of 20 kN/m over the left half of the span together with a point load of 100 kN at the right quarter span point. Find the reactions at the supports, normal thrust and shear at a section 10m from left support?



from FBD of ACB: $\sum M_B = 0$

$$V_A \times 40 - 20 \times 20 \times 30 - 100 \times 10 = 0$$

$$\boxed{V_A = 325 \text{ kN}}$$

$$\sum F_y = 0 \quad V_A + V_B = 20 \times 20 + 100$$

$$\boxed{V_B = 175 \text{ kN}}$$

from FBD of AC - applying $\sum M_C = 0$

$$V_A \times 20 - 20 \times 20 \times 10 - H \times 8 = 0$$

$$\boxed{H = 318.5 \text{ kN}}$$

from the property of circle $\frac{L}{2} \times \frac{L}{2} = h(\text{erf})$

$$20 \times 20 = 8(R-8)$$

$$\therefore R = 29 \text{ m}$$

for point D on the circle -

$$\sin \theta = \frac{10}{29} \Rightarrow \theta = 20.17^\circ$$

$$\begin{aligned} \text{Vertical shear, } V \text{ at } D &= V_A - 20 \times 10 \\ &= 325 - 200 = \underline{125 \text{ kN}} \end{aligned}$$

At 10m from left support:-

$$\begin{aligned} N (\text{Normal thrust}) &= V \sin \theta + H \cos \theta \\ &= 125 \sin 20.17^\circ + 325 \cos 20.17^\circ \\ &= \underline{336.437 \text{ kN}} \end{aligned}$$

$$\begin{aligned} Q (\text{Radial shear}) &= V \cos \theta - H \sin \theta \\ &= 125 \cos 20.17^\circ - 325 \sin 20.17^\circ \\ &= \underline{9.575 \text{ kN}} \end{aligned}$$

A circular arch to span 25m with a central rise 5m is hinged at the crown and springing. It carries a point load of 100kN at 6m from the left support calculate reactions at the supports and at crown. Also calc. moment at 5m from left support?

$$\begin{aligned} V_A &= 26 \text{ kN} & V_B &= 244 \text{ kN} & H &= 60 \text{ kN} \\ R &= 18.125 \text{ m} & \theta &= 24.43^\circ & y &= 3.375 \text{ m.} \\ M_D &= 177.5 \text{ kNm} \end{aligned}$$

prob 2
TSQ

Ans

③ prob:-

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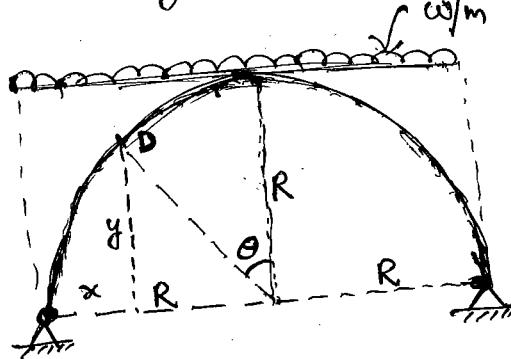
A Three Hinged Semi-circular arch of radius R carries a UDL of w/m , over its entire horizontal span. Determine the reactions of supports and maximum bending moment in the arch?

Sol:-

$$V_A = V_B = \frac{wR}{2} = wR$$

$$H \times R - V_A \times R + wR \cdot \frac{R}{2} = 0$$

$$H = \frac{wR}{2}$$



$$x = R - R \sin \theta \quad y = R \cos \theta$$

$$M_x = V_A \cdot x - Hy - \frac{w x^2}{2}$$

$$M_x = \frac{wR^2}{2} (1 - \cos \theta - \sin^2 \theta)$$

$$\frac{dM_x}{d\theta} = \sin \theta - 2 \sin \theta \cos \theta = 0$$

$$\sin \theta (1 - 2 \cos \theta) = 0$$

$$\sin \theta = 0 \Rightarrow \theta = 0^\circ; \text{ point 'C'}$$

$$1 - 2 \cos \theta = 0 \Rightarrow \cos \theta = \frac{1}{2} \quad \theta = 60^\circ$$

$$\therefore x = R - R \sin 60^\circ \quad ; \quad M_{\max} = - \frac{wR^2}{8}$$

* "funicular" is derived from the latin word "rope".

* A funicular structure is said to be a structure which can achieve a mechanism of a right form (shape/geometry) corresponding to the applied load.

Prob: ② A three hinged parabolic arch hinged at the support and at the crown has a span of 24m and a central rise of 4m. It carries a point load of 50 kN at 18m from left support and a UDL of 30 kN/m over the left half portion. Determine the moment, thrust and radial shear at a section 6m from left support?

$$\sum M_B = 0 : V_A = 282.5 \text{ KN}$$

$$\sum F_y = 0 \quad V_B = 127.5 \text{ KN}$$

$$\sum M_c = 0$$

$$H = 307.5 \text{ KN} \quad y = \frac{4hx(L-x)}{L^2} = 3 \text{ m}$$

$$\text{at } 6\text{m}, \quad BM = 232.5 \text{ KNm}$$

$$y = \frac{4hx(L-x)}{L^2} \Rightarrow \theta = \frac{4hxL}{L^2} - \frac{4hx^2}{L^2} = \frac{4hx}{L} - \frac{4hx^2}{L^2}$$

$$\frac{dy}{dx} = \frac{4h}{L} - \frac{8hx}{L^2} \Rightarrow \text{at } 6\text{m}, \frac{dy}{dx} = C = \tan \theta \quad \theta = 18.435^\circ$$

$$N = 324.133 \text{ KN} ; \quad Q = 0$$

Prob: ③ Show that the parabolic shape is a funicular shape for a three hinged arch subjected to a UDL over the entire span?

$$V_A = V_B = \frac{wL}{2}$$

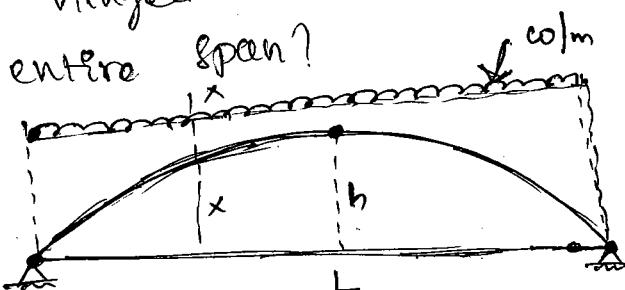
$$H = \frac{wL^2}{8h}$$

$$y = \frac{4hx(L-x)}{L^2} \quad M_x = \frac{wL}{2}x - \frac{wL^2}{8h}(y) - \frac{wx^2}{2}$$

$$= \frac{wL}{2}x - \frac{wL^2}{8h} \left(\frac{4hx}{L} - \frac{4hx^2}{L^2} \right) - \frac{wx^2}{2}$$

$$= \frac{wL}{2}x - \frac{wx^2}{2h} + \frac{wx^2}{2h} - \frac{wx^2}{2}$$

= 0



$$BM_x = 0$$

Two Hinged Arches

- * Two hinged arches are indeterminate structures.
- * They have degree of indeterminacy as One.
- * Commonly used geometry for two hinged arches are circular and parabolic.
- * Horizontal force 'H' may be taken as the redundant force. This can be found by theorem of Castigliano (8) unit load method.
- * Arches are structural members curved in elevation used to support heavy loads on large spans.

From Castigliano's first theorem:-

$$M_a = M' - Hy \quad (\because M' = \text{beam moment})$$

$$U = \int \frac{M_a^2}{2EI} ds$$

$$\frac{\partial U}{\partial H} = 0 \Rightarrow \int M_a \left(\frac{\partial M_a}{\partial H} \right) \frac{ds}{EI} = 0$$

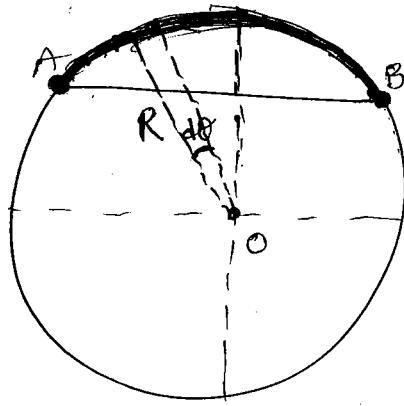
$$\int (M' - Hy)(-y) \frac{ds}{EI} = 0$$

$$\int M'y \frac{ds}{EI} = \int \frac{Hy^2}{EI} ds$$

$$\therefore H = \frac{\int M'y \frac{ds}{EI}}{\int y^2 \frac{ds}{EI}}$$

$$ds = R d\theta$$

$$y = h - R(1 - \cos\theta)$$



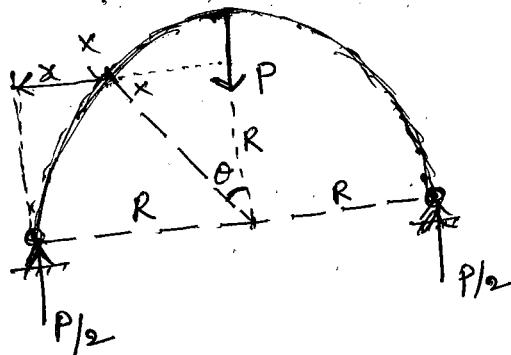
prob. ①

Determine horizontal thrust developed in a semi-circular arch of radius R subjected to a concentrated load P at the crown?

std:

$$H = \frac{\int M'y \, ds}{\int y^2 \, ds}$$

$$M' = \frac{P\alpha}{2} = \frac{PR(1 - \sin\theta)}{2}$$



$$\begin{aligned} \int M'y \, ds &= \int \left[\frac{PR}{2} (1 - \sin\theta) \right] (R \cos\theta) (R d\theta) \\ &= \frac{PR^3}{2} \int_0^{\pi/2} (1 - \sin\theta) \cos\theta \, d\theta \end{aligned}$$

$$\begin{aligned} \text{let } 1 - \sin\theta &= t ; \quad -\cos\theta \frac{d\theta}{dt} = dt \\ &= PR^3 \int_0^{\pi/2} t (-dt) = -PR^3 \left(\frac{t^2}{2} \right) = -PR^3 \cdot \left(\frac{(t^2)^0}{2} \right) \\ &= -PR^3 \left[0 - \frac{1}{2} \right] = \frac{PR^3}{2} \end{aligned}$$

$$\begin{aligned} \int y^2 \, ds &= \int (R \cos\theta)^2 R d\theta = \int_0^{\pi/2} R^2 \left(\frac{1 + \cos^2\theta}{2} \right) d\theta \\ &= R^2 \left[(\theta)_0^{\pi/2} + \left(\frac{\sin 2\theta}{2} \right)_0^{\pi/2} \right] \\ &= R^2 \left[\frac{\pi}{2} + 0 \right] = \frac{\pi R^3}{2} \end{aligned}$$

$$\therefore H = \frac{\left(\frac{PR^3}{2} \right)}{\left(\frac{\pi R^3}{2} \right)}$$

$$H = \frac{P}{\pi}$$

(neglecting self weight of arch).

Q prob:

Determine horizontal thrust in a semi circular arch subjected to UDL w/m over its horizontal span?

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soln

$$x = R(1 - \sin\theta)$$

$$y = R \cos\theta$$

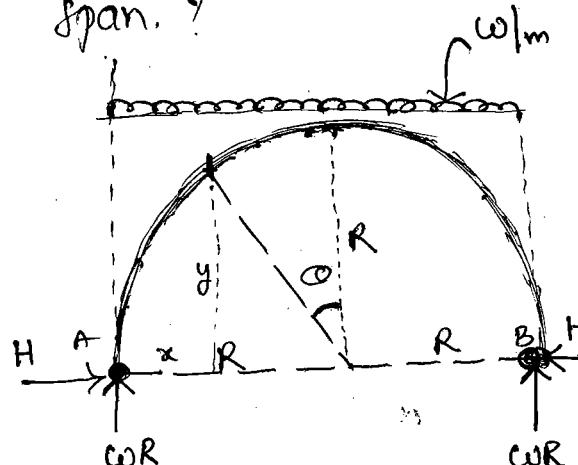
$$ds = R d\theta$$

$$M' = CORx - \frac{w x^2}{2}$$

$$= COR \cdot R(1 - \sin\theta) - \frac{COR^2(1 - \sin\theta)^2}{2}$$

$$= \frac{COR^2}{2} [2 - 2 \sin\theta - 1 + \sin^2\theta + 2 \sin\theta]$$

$$= \frac{COR^2}{2} [\cos^2\theta]$$



$$\therefore t = \sin\theta \\ dt = \cos\theta d\theta \rightarrow \left(\frac{t^3}{3}\right)$$

$$\int M' y ds = \int \frac{COR^2}{2} (\cos^2\theta) \times R \cos\theta R d\theta$$

$$= \frac{COR^4}{2} \int_0^{\pi/2} (1 - \sin^2\theta) \cos\theta d\theta$$

$$= COR^4 \left[\int_0^{\pi/2} \cos\theta d\theta - \int_0^{\pi/2} \sin^2\theta \cos\theta d\theta \right]$$

$$= COR^4 \left[\left(\sin\theta\right)_0^{\pi/2} - \left(\frac{t^3}{3}\right)_0^1 \right]$$

$$= COR^4 \left[1 - \frac{1}{3} \right] = \frac{2}{3} COR^4$$

$$\int y^2 ds = \frac{\pi}{2}(R^3) \Rightarrow \therefore H = \frac{(2COR^4)}{\frac{\pi R^3}{2}}$$

$$H = \frac{4 COR}{3 \pi}$$

prob.

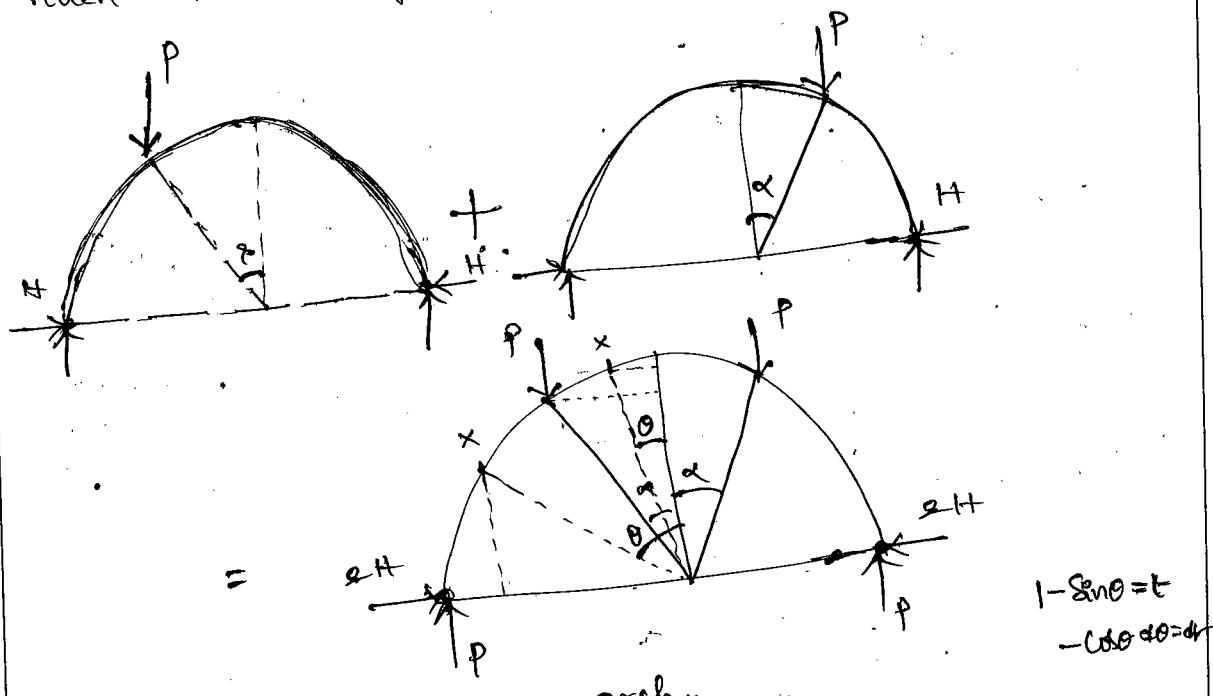
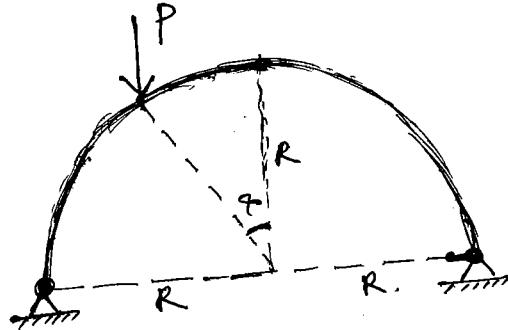
Determine horizontal reaction in a semi circular two hinged arch when a load P acts at a point D .

Ans:-

To make the given problem as symmetric loading,

add a force P at an angle α to the right side

then the horizontal thrust developed is $2H$.



$$1 - \sin\theta = t$$

$$-\cos\theta d\theta = dt$$

Now solving symmetric arch.

$$\text{for } \theta = 0 \text{ to } \alpha \quad M' = Px - P(R \sin\theta - R \sin\alpha)$$

$$M' = P(R - R \sin\alpha) - PR(\sin\alpha - \sin\theta)$$

$$= PR - PR\sin\alpha - PR\sin\theta + PR\sin\alpha = PR(1 - \sin\theta)$$

$$= PR - PR\sin\alpha - PR\sin\theta + PR\sin\alpha = PR(1 - \sin\theta)$$

$$\text{for } \theta = \alpha \text{ to } \pi/2 ; \quad M' = Px = P \times R(1 - \sin\theta)$$

$$\int M'y \, d\theta = \int_0^\alpha PR(1 - \sin\theta) R \cos\theta \, R d\theta + \int_\alpha^{\pi/2} PR(1 - \sin\theta) R \cos\theta \, R d\theta$$

$$= 2PR^3(1 - \sin\alpha)(\sin\theta)_0^\alpha + 2PR^3 \left(-\frac{1}{2} \right)$$

$$= 2PR^3(1 - \sin\alpha)\sin\alpha + 2PR^3 \left(-\frac{(1 - \sin\theta)^2}{2} \right)_\alpha^{\pi/2}$$

$$= \alpha PR^3 (\sin\alpha - \sin^2\alpha) + PR^3 (1 - \sin\alpha)$$

$$= \alpha PR^3 \sin\alpha - \alpha PR^3 \sin^2\alpha + PR^3 (1 + \sin^2\alpha - 2\sin\alpha)$$

$$= 2PR^3 \sin\alpha - 2PR^3 \sin^2\alpha + PR^3 + PR^3 \sin\alpha - 2PR^3 \sin\alpha$$

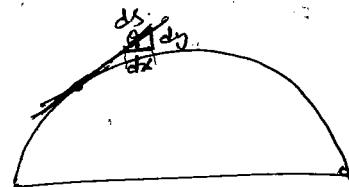
$$= PR^3 - PR^3 \sin^2\alpha = PR^3 (1 - \sin^2\alpha)$$

$$\int y^2 ds = \frac{\pi R^3}{2} \Rightarrow \text{HT} = \frac{PR^3 (1 - \sin^2\alpha) \times 2}{\pi R^3}$$

$$H = \frac{P \cos^2\alpha}{\pi}$$

Two Hinged parabolic Archest $H = \frac{\int M'y ds}{\int y^2 ds}$

$$H = \frac{\int M'y \left(\frac{da}{\cos\theta} \right)}{\int y^2 \frac{da}{\cos\theta}}$$



$$\cos\theta = \frac{dx}{ds}$$

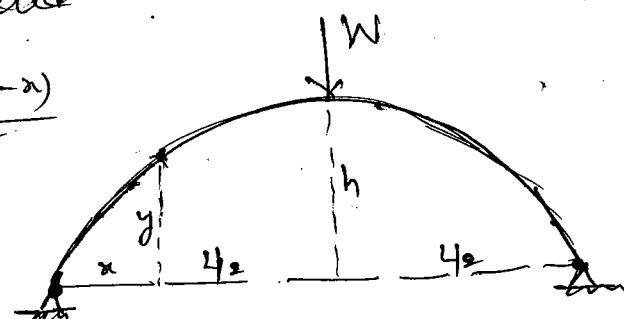
$$\therefore H = \frac{\int M'y da}{\int y^2 da}$$

$$da = \frac{da}{\cos\theta}$$

Prob Determine the horizontal thrust in a two hinged parabolic arch subjected to a point load at the crown?

Sol

$$M = \frac{Wx}{2} : y = \frac{4ha(L-x)}{L^2}$$



$$\therefore \int M'y da$$

$$= 2 \int_0^{L/2} \frac{Wa}{2} \left(\frac{4ha}{L^2} \right) (L-x) da = \frac{5 Whd^2}{48}$$

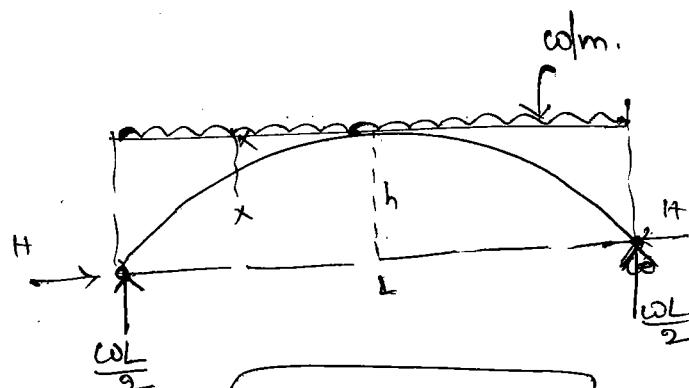
$$\int y^2 da = \frac{8}{15} Lh^2$$

$$\therefore H = \frac{25}{128} \frac{WL}{h}$$

Determine the horizontal thrust in a two hinged parabolic arch subjected to a UDL of $\omega \text{ N/m}$ on its horizontal span?

$$H = \frac{\int M y \, dx}{\int y^2 \, dx}$$

numi- $\int M y \, dx$



$$= \int \left(\frac{\omega L x}{2} - \frac{\omega x^2}{2} \right) \frac{4h^2(L-x)}{L^2} \, dx \quad M = \frac{\omega L x}{2} - \frac{\omega x^2}{2}$$

$$= \frac{4h\omega}{L^2} \int \left(\frac{x}{2} - \frac{x^2}{2} \right) (Lx - x^2) \, dx$$

$$= \frac{4h\omega}{2L^2} \int (Lx - x^2)^2 \, dx$$

$$= \frac{4h\omega}{2L^2} \int_0^L \left(L^2 x^2 + x^4 - 2Lx^3 \right) \, dx$$

$$= \frac{4h\omega}{2L^2} \left[L^2 \left(\frac{x^3}{3} \right)_0^L + \left(\frac{x^5}{5} \right)_0^L - 2L \left(\frac{x^4}{4} \right)_0^L \right]$$

$$= \frac{4h\omega L^5}{2L^2} \left[\frac{1}{3} + \frac{1}{5} - 2 \times \frac{1}{4} \right] \quad \left(\frac{10+6-15}{30} = \frac{1}{10} \right)$$

$$= 2h\omega L^3 \left(\frac{1}{3} + \frac{1}{5} - \frac{1}{2} \right)$$

$$= \frac{2h\omega L^3}{30} = \frac{h\omega L^3}{15}$$

$$\text{Open} = \frac{8Lh^2}{15}$$

$$H = \frac{K \omega L^3}{15} / \frac{8h^2}{15} = \frac{\omega L^2}{8h}$$

prob. Determine horizontal thrust developed in a two hinged parabolic arch subjected to a point load w at the crown?

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Sols

$$H = \frac{5 w h L^2}{48} / \frac{.8 L h^2}{15} = \frac{25 w L}{128 h}$$

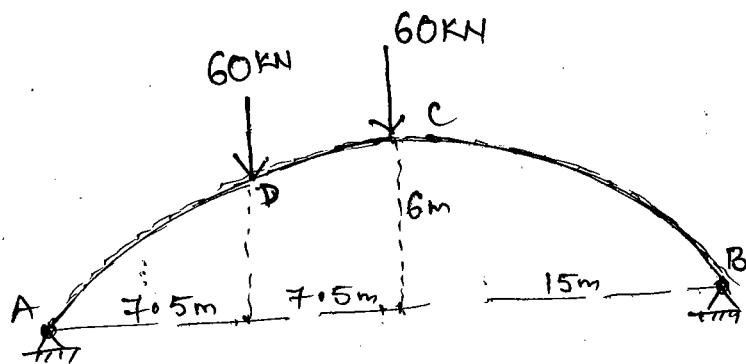
prob:

Two-hinged parabolic arch of Span 30m and rise 6m carries two point loads each 60 kN acting at 7.5m and 15m from left. Determine horizontal thrust and maximum positive moments in the arch?

Sols:-

$$V_A = 75 \text{ kN}$$

$$V_B = 45 \text{ kN}$$



$$\text{for } x = 0 \text{ to } 7.5 : M = 75x$$

$$x = 7.5 \text{ to } 15 : M = 75x - 60(x - 7.5) = 15x + 450$$

$$x = 0 \text{ to } 15 \text{ from B} : M = 45x$$

$$y = \frac{4h^2(L-x)}{L^2} = \frac{4x(6)(30-x)}{30^2}$$

$$\int M y \, dx = 57498.87 ; \int y^2 \, dx = 576$$

$$\therefore H = 100.345 \text{ kN}$$

$$BM_D = 110.95 \text{ kNm}, BM_C = 42.93 \text{ kNm.}$$

$$\text{max shear } BM = 110.95 \text{ kNm}$$

$$M_n = 2(5x - 100) \cdot 345 \text{ Nm}$$

$$\frac{dM_n}{dx} = 0 \Rightarrow x = 6.591 \text{ m.}$$

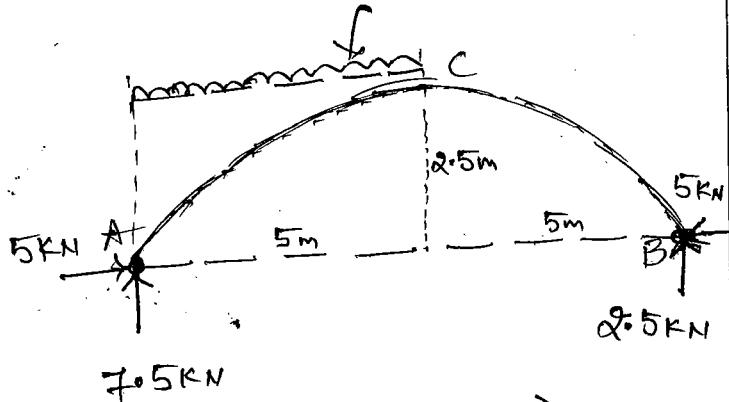
$$M_{\min} = -116.25 \text{ KNm.}$$

Nov-15 Prob:

A two hinged parabolic arch has a span of 10m and a central rise of 2.5m. It is loaded with UDL of 2 KN/m over a half of the span from left support. Determine end reactions, horizontal thrust, max & min. BM in the arch?

$$H = \frac{\int M' y \, dx}{\int y^2 \, dx}$$

$$= \frac{166.665}{33.334} = 5 \text{ KN}$$



AC: $M_x = 2.5x - x^2 - 5 \left(\frac{4 \times 2.5x (10-x)}{100} \right)$

$$= 2.5x - 0.5x^2 \quad M_{\max} = 3.125 \text{ KNm.}$$

$$\frac{dM_n}{dx} = 0 \Rightarrow x = 2.5 \text{ m.}$$

$$M_n = 2.5x - 5 \left(\frac{4 \times 2.5x (10-x)}{100} \right)$$

BCF: $\frac{dM_n}{dx} = 0 \Rightarrow x = 2.5 \text{ m.}$

$$M_{\max} = -3.125 \text{ KNm.}$$

$$M_c = 0$$

Effect of temperature :- Let temperature of an arch is increased by $t^{\circ}\text{C}$. Supports exert pressure on the arch.

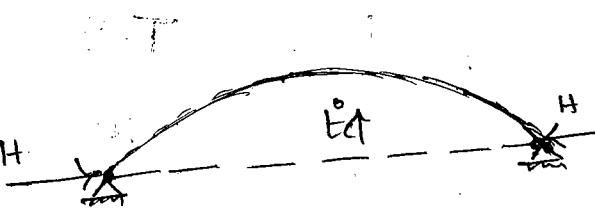
$$M_a = -Hy$$

$$U = \int \frac{M_n^2 dx}{2EI}$$

$$\frac{\partial U}{\partial H} = L_{axt} = \frac{\partial U}{\partial H} \left(\frac{M_n^2 dx}{2EI} \right) = \frac{1}{2EI} \int \partial (-Hy)^2 dx$$

$$= \frac{1}{2EI} \int \partial (-Hy)(-y) dx = \int \frac{Hy^2 dx}{EI}$$

$$L_{axt} = H \int \frac{y^2 dx}{EI} \Rightarrow H_t = \frac{L_{axt}}{\int \frac{y^2 dx}{EI}}$$



Rib shortening :- (Due to normal thrust)

$$M_n = M' - Hy$$

$$N = V \sin \theta + H \cos \theta$$

$$U = \int \frac{N_n^2}{2AE} ds \Rightarrow \frac{\partial U}{\partial H} = 0$$

$$\therefore \int \frac{(M' - Hy)(-y)}{EI} ds + \int \frac{(V \sin \theta + H \cos \theta) \cos \theta ds}{AE} = 0$$

$$\Rightarrow - \int \frac{M' y ds}{EI} + \int \frac{Hy^2 ds}{EI} + \int \frac{H \cos^2 \theta ds}{AE} -$$

neglecting shear

$$+ \int \cancel{H \sin \theta \cos \theta ds} = 0$$

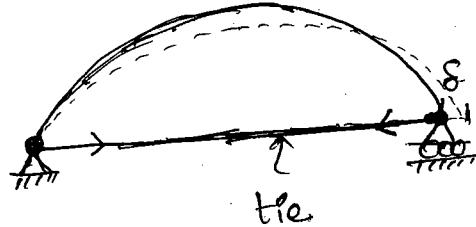
$$H = \frac{\left(\int \frac{M' y ds}{EI} \right)}{\int \frac{y^2 ds}{EI} + \int \frac{\cos^2 \theta ds}{AE}}$$

Pied Arch: $\delta = \frac{HL}{AE}$

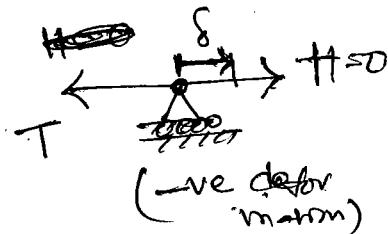
$$M_x = M' - HY$$

$$= M' - HY$$

$$\frac{\partial U}{\partial H} = \frac{HL}{AE} \Rightarrow \frac{\partial \left(\frac{M_x^2 dm}{2EI} \right)}{\partial H} = \left(-\frac{HL}{AE} \right)$$



$$\int \frac{\partial (M' - HY) E_y dm}{\partial B \delta} = \frac{HL}{AB}$$



$$-\int \frac{M' y dm}{B \delta} + \int \frac{HY^2 dm}{B \delta} = -\frac{HL}{AE}$$

$$\int \frac{HY^2 dm}{EI} + \frac{HL}{AE} = \int \frac{M' y dm}{EI}$$

$$H = \frac{\int \frac{M' y dm}{EI}}{\left(\int \frac{Y^2 dm}{EI} + \frac{L}{AE} \right)}$$

Fixed Arch :- (Hingedless arch) Analytic method

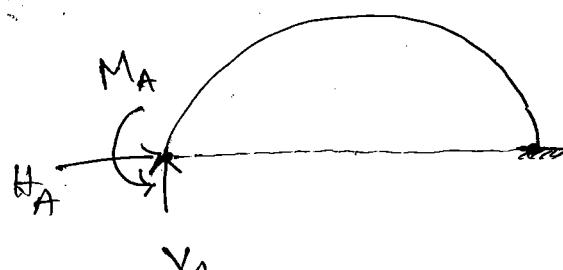
(i) CDM (ii) Elastic centre method

$$\Delta V, \Delta H, \Theta_A = 0$$

Let M' be the moment due to given load,

m_1, m_2 and M_3 be the

moments due to unit loading in the directions of H_A, V_A & M_A .



(i) due to given loading, $\Delta V_{eo} = \int \frac{m_m ds}{EI}$

SA-II
I-II

$$\Delta H_{AO} = \int \frac{M^l m_2 ds}{EI} ; \quad \Delta \Theta_{AO} = \int \frac{M^l m_3 ds}{EI}$$

$$\text{iii) Due to } V_A \text{ alone, } \Delta V_{A1} = \int (V_A^2) m_1 \frac{dk}{E^2}$$

$$\Delta H_{A1} = \int V_{A2} \cdot m_2 \frac{dy}{Ee} ; \quad \Delta \Theta_{A1} = \int V_{A2} \cdot m_3 \frac{dy}{Ee}$$

$$\text{iii) Due to } H_A \text{ alone} \quad \Delta H_{\text{se}} = \int H_A y \cdot m \frac{ds}{E.S}$$

$$\Delta H_{A2} = \int (H_A \cdot y) m_2 \frac{dy}{E.P} ; \quad \Delta \Theta_{A2} = \int H_A \cdot y \cdot m_3 \frac{dy}{E.P}$$

(iv) Due to M_A alone $\Delta V_{AS} = \int M_A \cdot m_i \frac{dy}{ES}$

$$\Delta H_{AB} = \int M_A \cdot m_B \frac{dk}{EI} ; \quad \Delta \theta_{AB} = \int M_A \cdot m_3 \frac{dk}{EI}$$

consistency conditions:-

$$\Delta V^{\infty} ; \quad \Delta H_A^{\infty} \quad \Delta G_A^{\infty}$$

$$\text{eqn } ① \Rightarrow \Delta V_{A0} + \Delta V_{A1} + \Delta V_{A2} + \Delta V_{A3} = 0$$

$\text{eq}^n \circledcirc \Rightarrow$

e.g. ③ \Rightarrow

Solving eqns ①, ② & ③

we get $V_A, H_A, \epsilon^M A$

PROBLEMS (JNTUH)

Oct-16

1. (a) How are arches classified based on shape and end conditions? (8M)
- (b) State and prove Eddy's theorem & (8M)
2. A ~~three hinged parabolic~~ Derive the expression for normal thrust, radial shear and horizontal thrust for a two hinged arch? (16M)
3. what is the effect of rib shortening on two hinged arch? (3M)
4. what is the effect of temperature on two hinged arch? (3M)
5. find the horizontal thrust of a two hinged semi circular arch of radius R carries a concentrated load of W ? (4M)
6. A two-hinged parabolic arch of span 14m and rise 2.2m, carries two concentrated loads of 30KN each placed at the crown and at the right quarter span section. Find the horizontal thrust and BM at loaded point? (15M)
7. A two hinged parabolic arch of span 35m and rise 5m carries a UDL of 40KN/m on the left half span. Determine horizontal thrust and BM at the crown? (15M)
8. A two hinged parabolic arch of span 30m and rise 6m carries a triangular load covering left half of the span, the intensity of load varying uniformly from 0 to 10KN/m. Evaluate the horizontal thrust?

Nov-15

May-15
(R10)

May-15
R10.
S

May-15

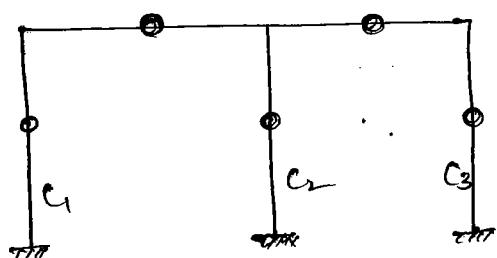
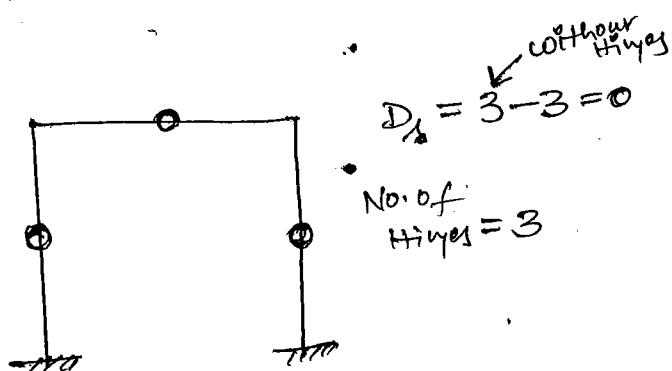
Lateral load analysis using Approximate Methods:-

- (i) portal method
- (ii) cantilever method
- (iii) factor method (not in the syllabus).
- (iv) for Vertical loads : Substitute frame method

(i) portal method:-

- Assumptions:-
1. point of contraflexure occurs at the middle of all members of frame
 2. Horizontal shear taken by each interior column is double of that taken by exterior column.

The above assumption will make the frame determinate and this method is proposed by Albert Smith in 1915.

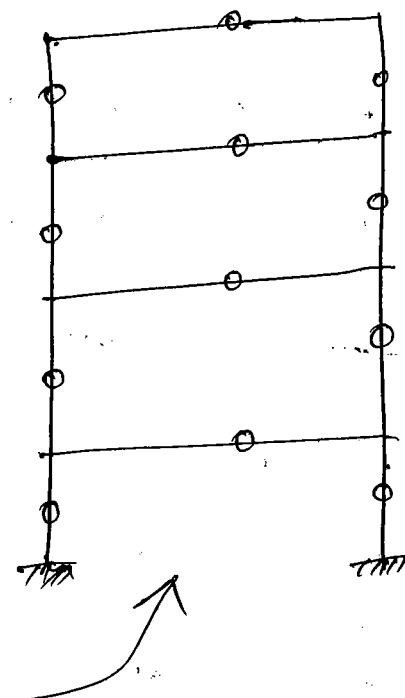
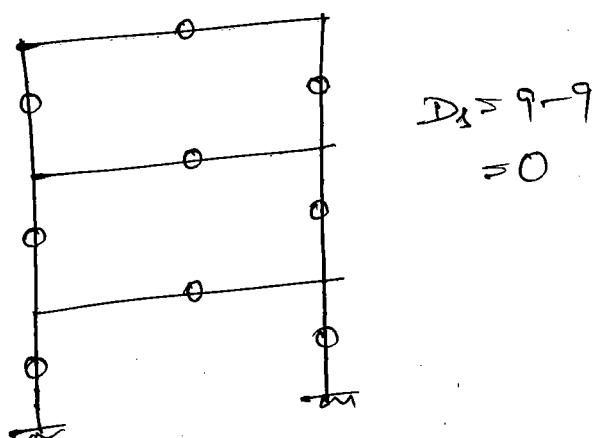
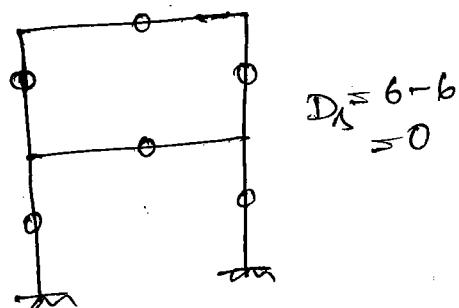
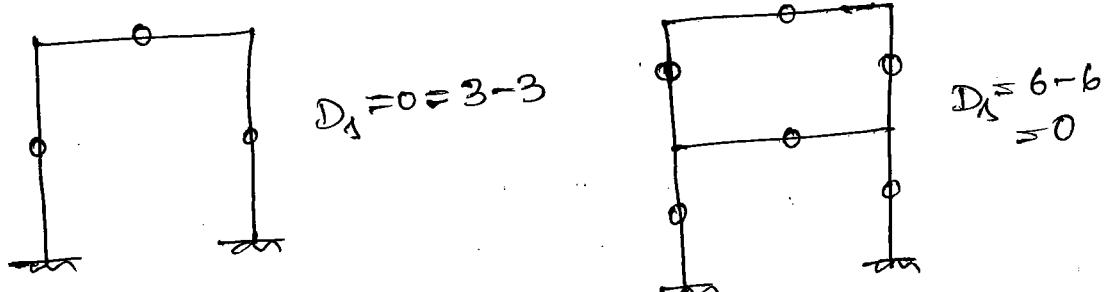
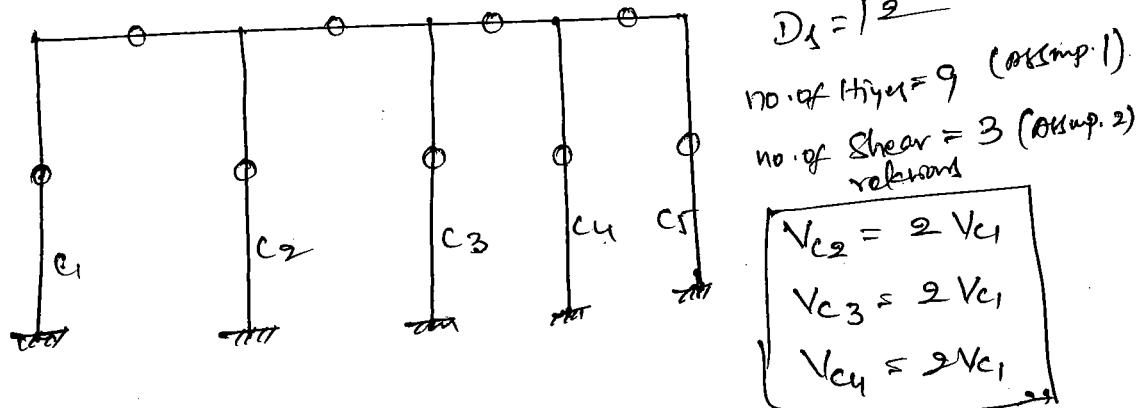
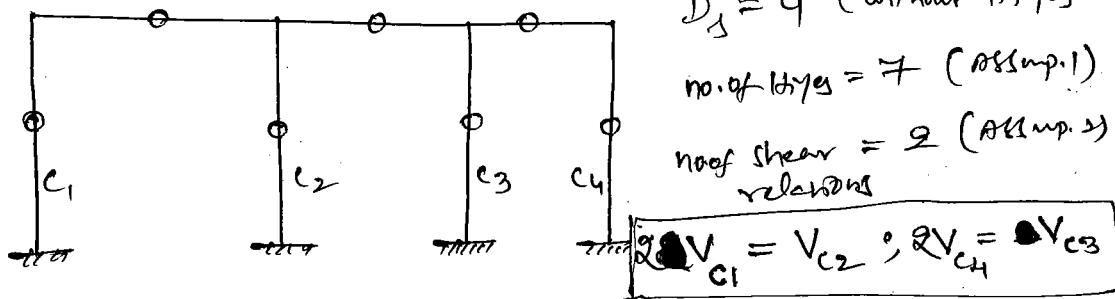


$$D_s = 6 \text{ (without Hinges)}$$

$$\text{No. of Hinges} = 5 \text{ (Assmp. 1)}$$

$$\text{No. of shear relations} = 1 \text{ (Assmp. 2)}$$

$$V_{C2} = 2 V_{C1}$$



$\underline{D_s = 12 - 12 = 0}$

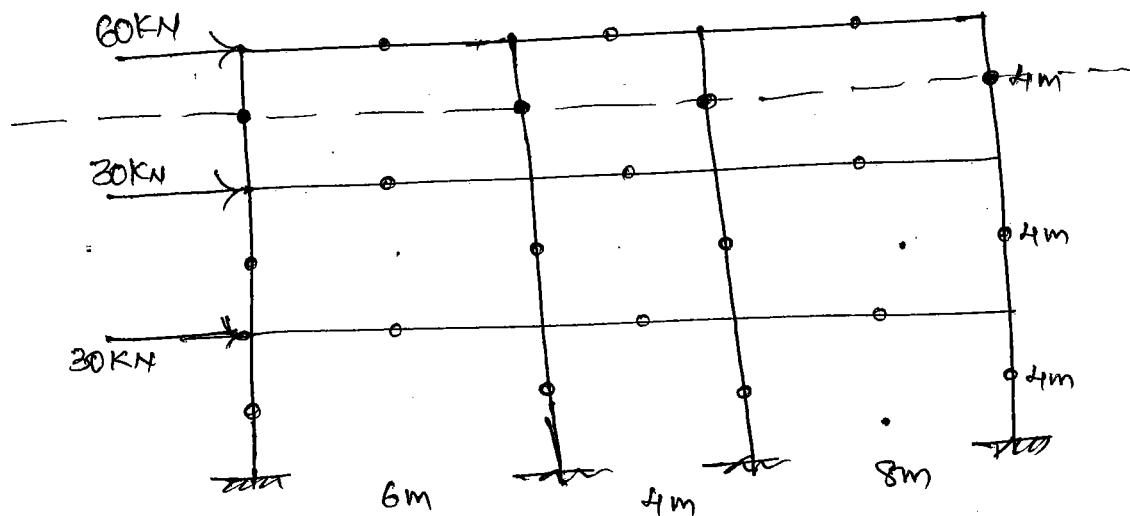
prob:

The following figure shows wind loads transferred to joints at A, D, G are 12 kN, 24 kN and 24 kN. Analyse the frame by portal method?

SA-II
II-@

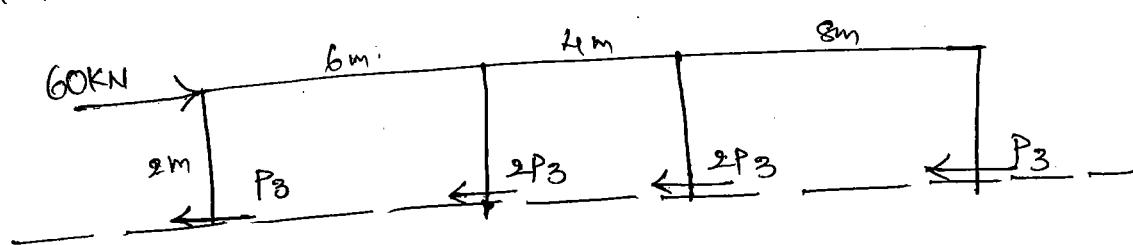
May-16
prob

Analyse the frame shown below by using portal Method?



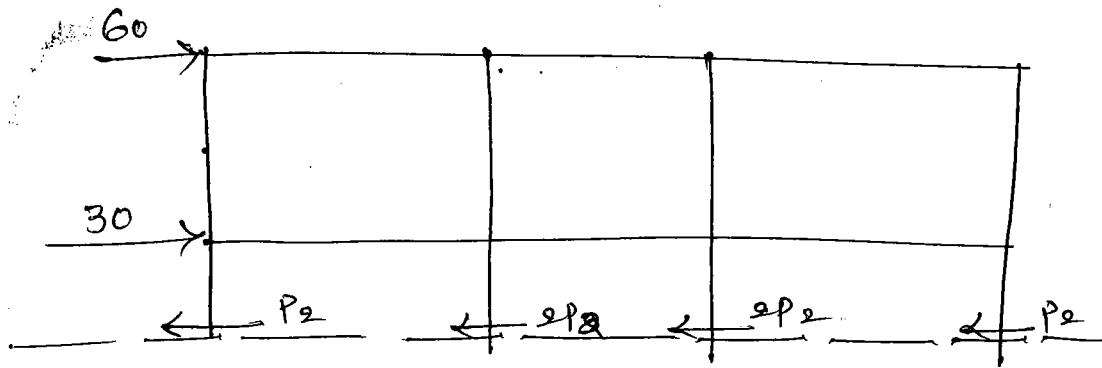
Step 1:- column shear:

Assumption:- Horizontal shear taken by each interior column is double that of exterior.



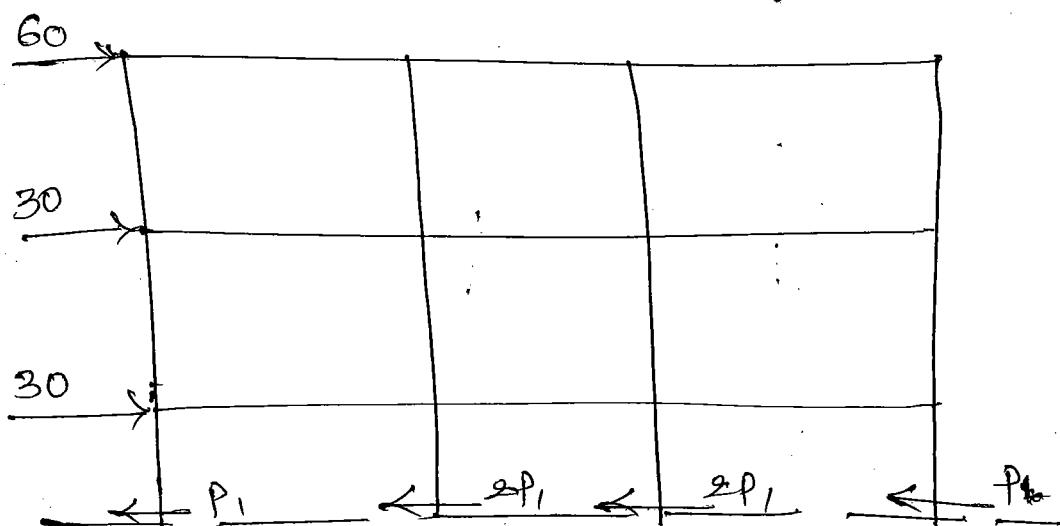
$$\begin{aligned} 60 &= P_3 + 2P_3 + 2P_3 + P_3 = \\ \therefore 60 &= 6P_3 \Rightarrow P_3 = 10 \text{ KN.} \end{aligned}$$

Taking section in the middle of storey at all storey levels we can calculate column shear



$$60 + 30 = P_2 + 2P_2 + 2P_2 + P_2$$

$$90 = 6P_2 \Rightarrow P_2 = 15 \text{ KN}$$



$$60 + 30 + 30 = P_1 + 2P_1 + 2P_1 + P_1$$

$$120 = 6P_1 \Rightarrow P_1 = 20 \text{ KN}$$

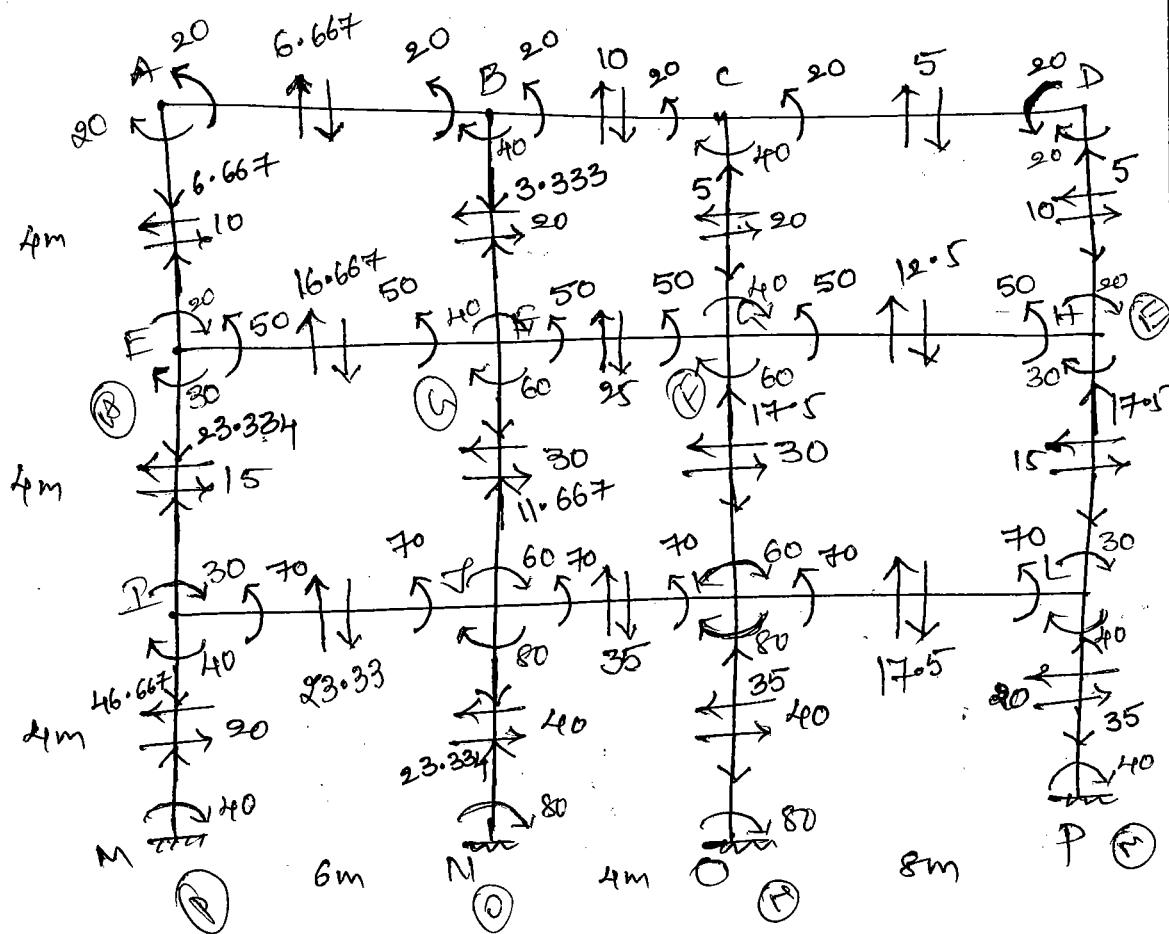
Hence, column shear in

$$3^{\text{rd}} \text{ Storey} = 10 \text{ KN}$$

$$2^{\text{nd}} \text{ "} = 15 \text{ KN}$$

$$1^{\text{st}} \text{ "} = 20 \text{ KN}$$

Step 2:- Column moments



$$\frac{\text{col AE}}{\text{at A}} = 10 \times 2 = 20 \text{ KNM} = \text{at E}$$

Uly column D11 \Rightarrow 40 kNm .

My column at B, $BM = 50 \times 2 = 100$
 column BF, at B, all the columns at all storey levels
 My for all the green box

Step 3:- Beam end moments (Indicated in green ink)

$$\text{Ans: } \text{end A} = M_{AE} + M_{AB} = 0$$

$$\text{Beam AB: } \text{end A} = M_{AE} + M_{AB}$$

$$\therefore M_{BA} = -20 \text{ KNM}$$

$$M_{PA} + M_{Re} + M_{DF} = 0$$

$$\rightarrow 20 + M_{BC} + \text{H}_2\text{O} \xrightarrow{50}$$

$$M_{BC} \leftarrow 20 \text{ kNm}$$

for Beam CDS-

$$M_{CB} + M_{CG} + M_{CD} \geq 0$$

My for all Beams

$$-20 + 40 + MCD \leq 50 \quad \text{in } MCD$$

Step 4:- Beam Shear

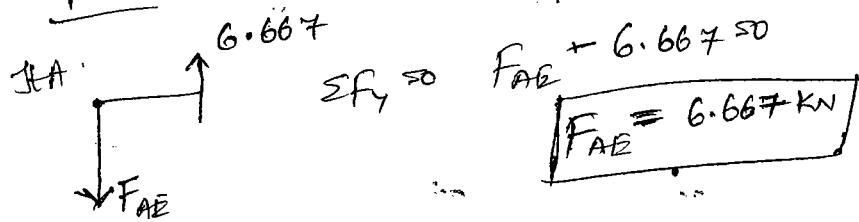
$$\text{AB} \perp \quad B.M = 20 = V_{AB} \times 3 \Rightarrow V_{AB} = 6.667 \text{ kN}$$

$$V_{BC} = \frac{20}{2} = 10 \text{ kN}$$

$$V_{CD} = \frac{20}{4} = 5 \text{ kN}$$

Similarly for all other beams.

Step 5:- Column axial force



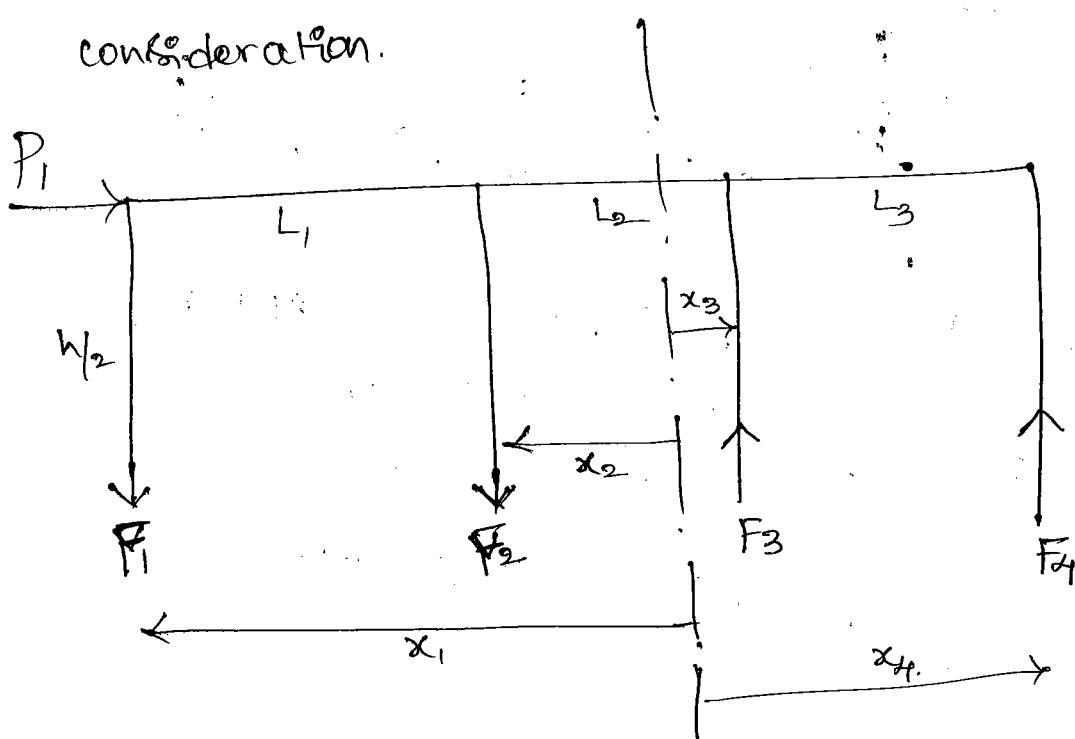
Similarly for all other columns.

prob: Analyse a portal frame of two storeys and two bay of 5m bay length each and height 5m. A horizontal force of 100 kN is applied at top storey and 60 kN is at lower storey. Use portal method?

Cantilever Method:-

Assumptions:- (1) there is a point of contraflexure at the centre of each member.

- (2) Intensity of axial stress in each column of a Storey is proportional to the distance of that column from C.G. of all columns of the storey under consideration.



$$\frac{(F_1)}{x_1} = \frac{(f_2)}{x_2} = \frac{(F_3)}{x_3} = \frac{(F_4)}{x_4} \quad \textcircled{1}$$

Taking moment about the point of contraflexure of the first column

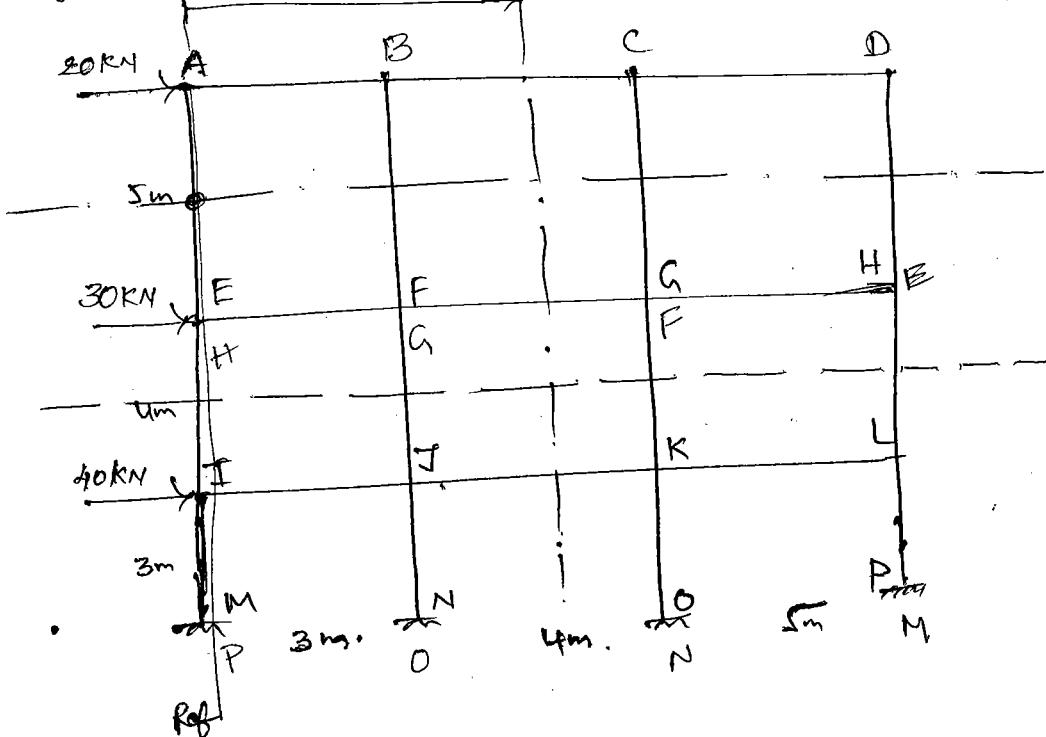
$$P_1 \left(\frac{h}{2} \right) = F_2 L_1 - F_3 (L_1 + L_2) - F_4 (L_1 + L_2 + L_3) \quad \textcircled{2}$$

using ① & ② we can find

$$\underline{f_1, f_2, f_3 \text{ and } f_4}$$

act 16
prob 1

Analyse the portal frame by Cantilever Method



Step 1:- Distance of C.G. of column AEI is given by

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{0 + a \times 3 + a \times 7 + a \times 12}{a + a + a + a} = \frac{22a}{4a} = 5.5\text{m}$$

Assumption

$$\frac{(F_{AE})}{5.5} = \frac{(F_{FB})}{2.5} = \frac{(F_{CG})}{1.5} = \frac{(F_{DH})}{6.5}$$

$$F_{AE} \times \frac{2.5}{5.5} = F_{FB} \quad \left(\frac{1.5}{5.5} \right) F_{AE} = F_{CG} \quad \left(\frac{6.5}{5.5} \right) F_{AE} = F_{DH}$$

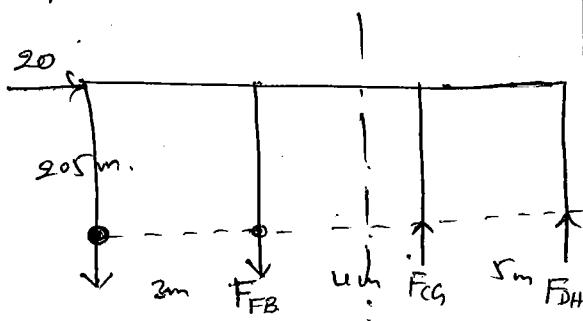
$$0.455 F_{AE} = F_{FB}$$

$$0.273 F_{AE} = F_{CG}$$

$$1.182 F_{AE} = F_{DH}$$

Pairing moment at mid point of AE,

$$20 \times 2.5 + F_{FB} \times 3 - F_{CG} \times 7 - F_{DH} \times 12 = 0$$



$$20 \times 2.5 + (0.455 F_{AE}) \times 3 - (0.273 F_{AE}) \times 7 \\ - (1.182 F_{AE}) \times 12 = 0$$

$$50 + [1.365 - 1.911 - 14.184] F_{AE} = 0$$

$$F_{AE} = \frac{50}{14.73} = 3.4 \text{ KN}$$

$$F_{FB} = 1.55 \text{ KN} ; F_{CG} = 0.93 ; F_{DH} = 4 \text{ KN}$$

Storey 2:-

$$\frac{F_{EI}}{5.5} = \frac{F_{FJ}}{2.5} = \frac{F_{AK}}{1.5} = \frac{F_{HL}}{6.5}$$

Taking moments about mid point of EI. Chm.

$$20 \times 7 + 30 \times 2 + F_{FJ} \times 3 - F_{AK} \times 7 - F_{HL} \times 12 = 0$$

$$20 \times 7 + 30 \times 2 + \left[\frac{2.5 \times 3}{5.5} - \frac{1.5 \times 7}{5.5} - \frac{6.5 \times 12}{5.5} \right] F_{EI} = 0$$

$$140 + 60 + [1.364 - 1.91 - 14.182] F_{EI} = 0$$

$$F_{EI} = \frac{200}{14.728} = 13.58 \text{ KN}$$

$$F_{FJ} = 6.17 \text{ KN}; F_{AK} = 3.4 \text{ KN}; F_{HL} = 16.05 \text{ KN}$$

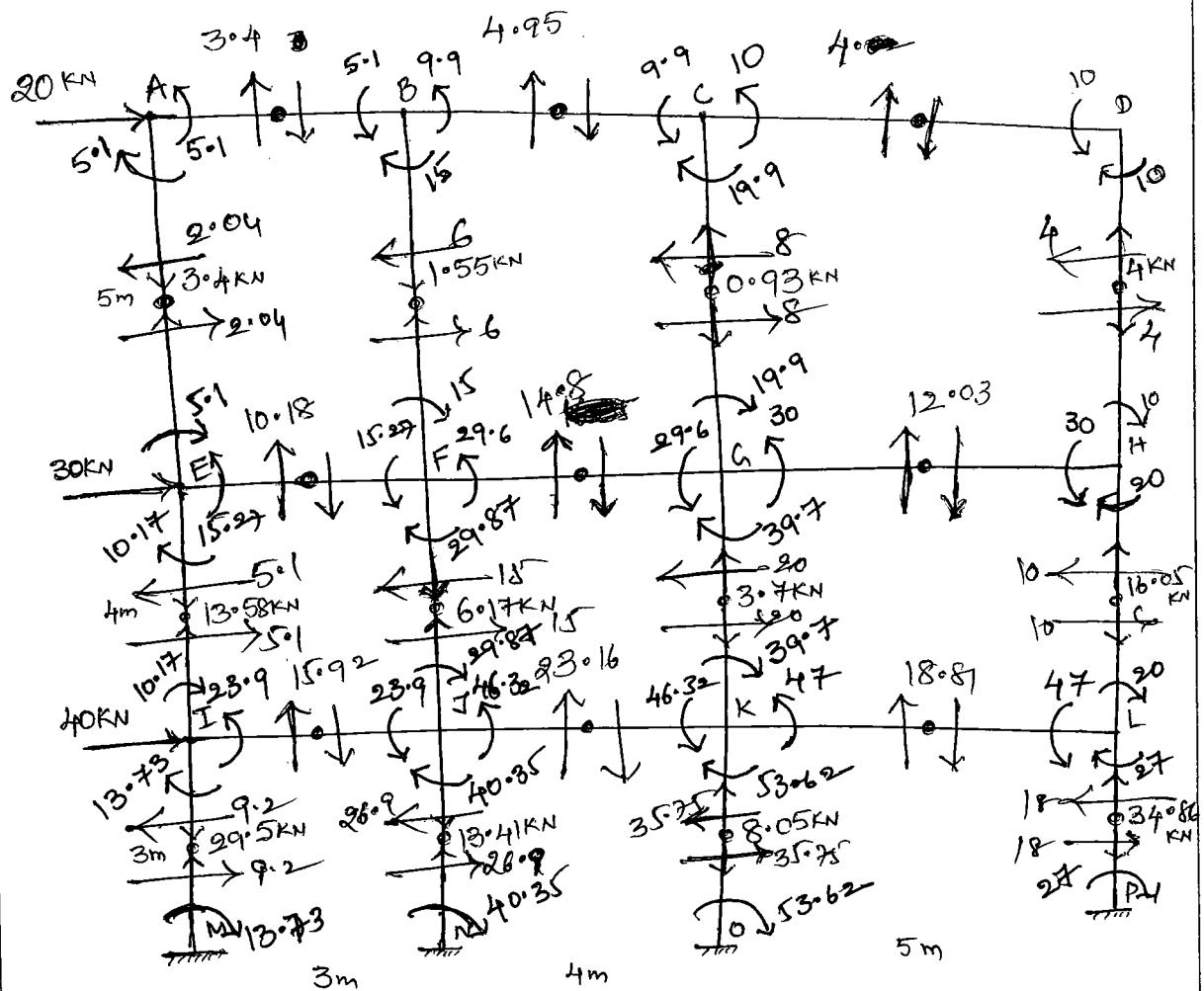
Storey 3:-

$$\frac{F_{IM}}{5.5} = \frac{F_{JN}}{2.5} = \frac{F_{KO}}{1.5} = \frac{F_{LP}}{6.5}$$

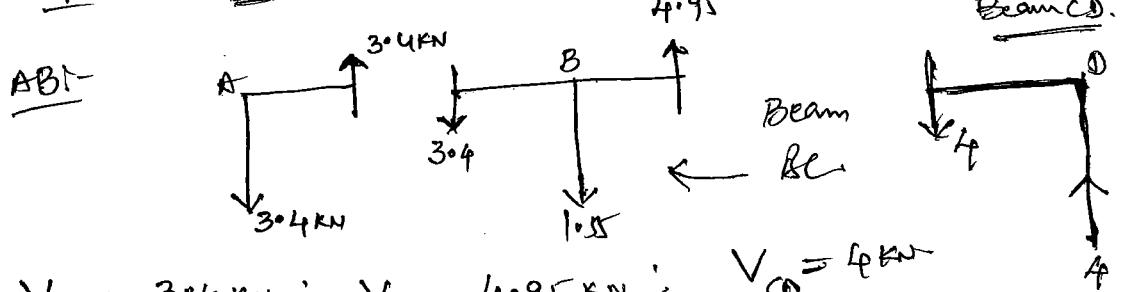
$$20 \times 10.5 + 30 \times 5.5 + 40 \times 1.5 = 14.728 F_{IM}$$

$$F_{IM} = 89.5 \text{ KN}$$

$$F_{JN} = 13.41 \text{ KN}; F_{KO} = 8.05 \text{ KN}; F_{LP} = 34.86 \text{ KN}$$



Step 2 :- Beam Shear



$$V_{AB} = 3.04 \text{ kN}; V_{BC} = 4.95 \text{ kN}; V_{CD} = 4 \text{ kN}$$

likewise, for all other beams.

Step 3 :- Beam end moments :-

$$M_{AB} = 3.04 \times 1.5 = 5.1 \text{ KNm} = M_{BA}$$

$$M_{BC} = M_{CD} = 4.95 \times 2 = 9.9 \text{ KNm}$$

$$M_{CD} = M_{DC} = 4 \times 2.5 = 10 \text{ KNm}$$

likewise, for all other beams.

Step 4:- Column moments:-

Column AE, $M_{AB} + M_{AE} = 0$
 $-5.1 + M_{AE} = 0 \Rightarrow M_{AE} = 5.1 \text{ kNm}$

at FB, $M_{BA} + M_{BC} + M_{BF} = 0$ $M_{BF} = 15 \text{ kNm}$
 $-5.1 + (-9.9) + M_{BF} = 0$

Do for all other columns.

Step 5:- Column Shear

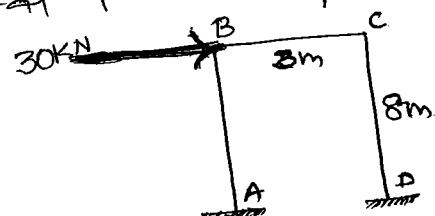
$V_{AE} = \frac{5.1}{2.5} = 2.04 \text{ kN}$ $V_{BF} = \frac{15}{2.5} = 6 \text{ kN}$

$V_{CH} = \frac{19.9}{2.5} = 8 \text{ kN}$ $V_{DH} = \frac{10}{2.5} = 4 \text{ kN}$

Do for all other columns.

May 15

Analyse the portal frame by using portal method?

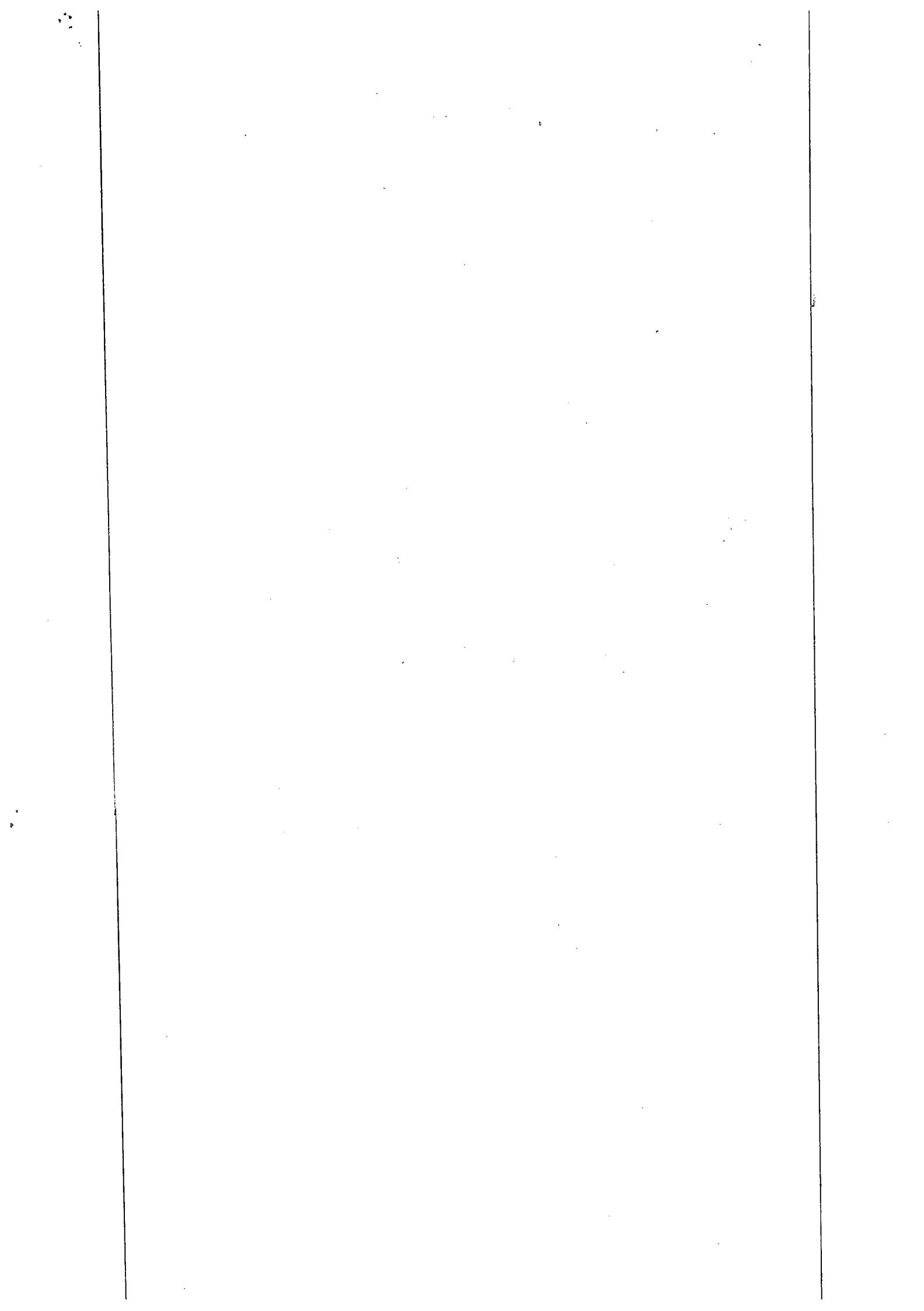


Nov 15

Explain the portal method for analysing a building frame subjected to horizontal forces?

Nov 15

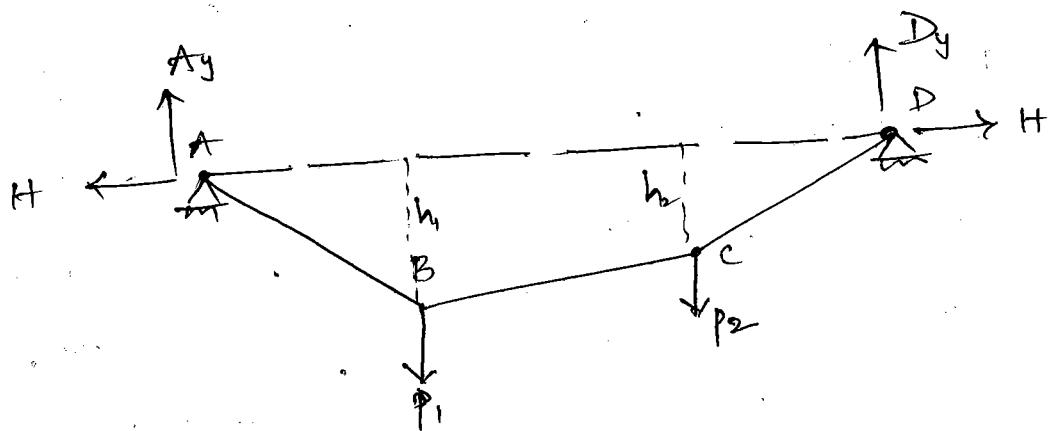
Write the steps involved in portal frame method?



Cable Structures and Suspension Bridges:-

Introduction:- Cables are flexible wire-like systems having no flexural stiffness (bendy) and they can carry only axial tension and no other type of force.

- Being fully flexible against bending the shape of a cable is determined by the external forces that are acting on the cable (fig. below)



- The shape of above cable is too depends on location and magnitude of P_1 & P_2 .

- A cable is unable to carry BM, SF, torsion or axial comp. Nevertheless, cables can be very effectively used in achieving long-span light-weight systems, such as bridges or roofs for large areas.

→ Two kinds of bridge structural systems where cables are used are the Suspension-cable system and cable-stayed system.

e.g.- Golden gate bridge, S.F., USA is suspension-cable type
ANZAC Bridge, Sydney, AUS. is cable-stayed

- cables are usually made of multiple strands of cold-drawn high-strength steel wires twisted together.
- Generally, they have strength four to five times that of structural steel and practically inextensible under operating loading conditions.
- Since cables carry only axial tension, full potential of the cable can be utilized in transferring forces. Therefore, cables are able to carry the same amount of force with a much smaller CFS compared to other structural systems.
- The high strength-to-weight ratio makes cables very useful where light-weight systems are needed. On the other hand, carrying a beam over a very long span could require a very large (and deep) CFS, and most of its potential will be used in carrying internal forces due to its own weight.

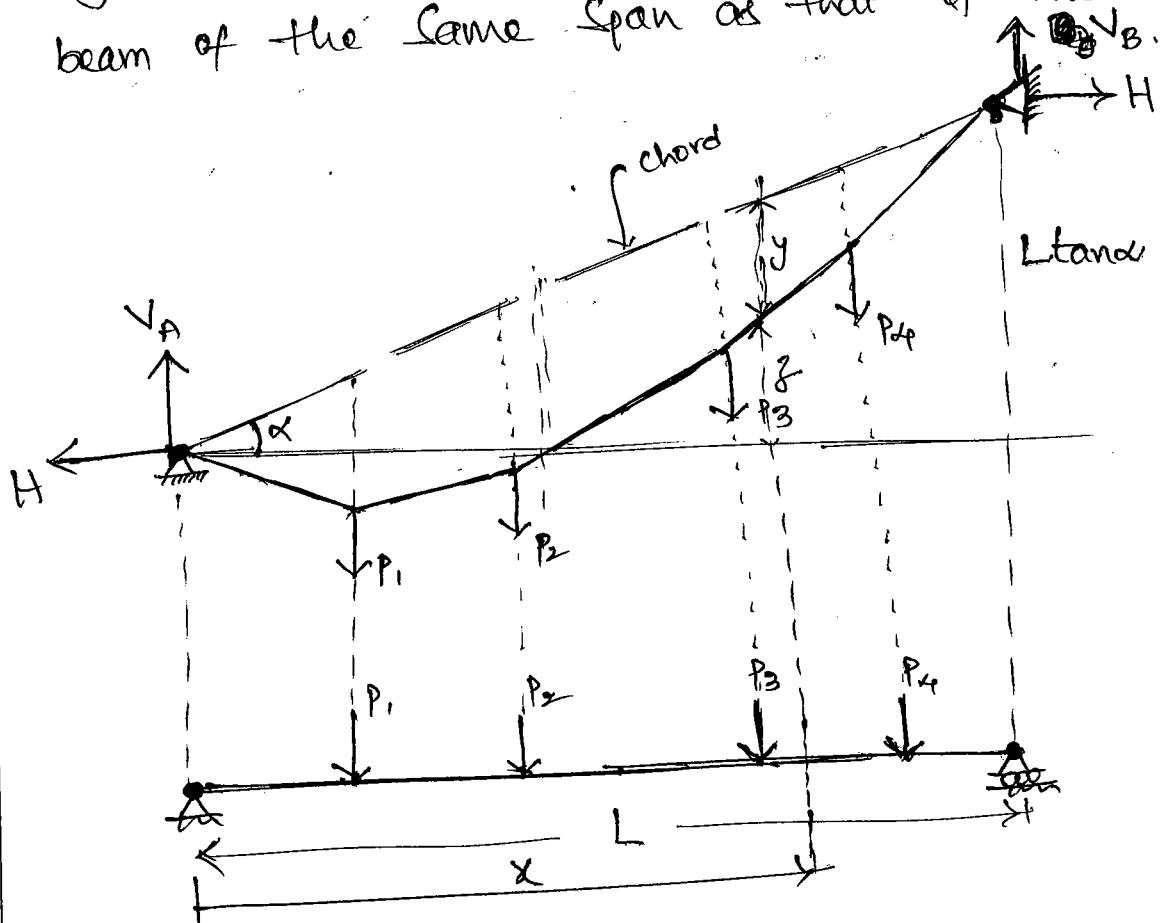
- If we use cables replacing this beam or in combination with a beam instead, a lighter structure could be required, whose self-weight could not add significantly to load effects
- The main disadvantage with cables is due to their flexible geometry. As the loading on a cable system changes (moving load) there can also be large change in the cable geometry and subsequently on forces acting in the cable. Unexpected forces may destabilize a cable system, causing excessive deformations. A designer should be very careful on this regard while designing a cable system, along with other issues such as, large forces at the anchors, large oscillations, etc.

e.g.: TACOMA NARROW BRIDGE failure

The general cable theorem:-

It helps us determine the shape of a cable supported at two ends when it is acted upon by vertical forces.

Statement : At any point on a cable acted upon by vertical loads, the product of the horizontal component of cable tension and the vertical distance from that point to the cable chord equals the moment which could occur at that section if the loads carried by the cable were acting on an simply-supported beam of the same span as that of the cable.



→ Consider cable AB, the line AB joining the two supports is known as the chord and the horizontal distance b/w the supports is span.

→ vertical distance b/w chord and the cable at any ck is known as the dip.

SA-II

III-③

→ moment equilibrium about B:-

$$V_A L + H L \tan\alpha = \sum M_{BP} \quad \text{--- (1)}$$

Since the cable is totally flexible against bending, BM at any ck is zero. By equating BM at x from A to zero,

$$V_A x + H y = \sum M_{xp} \quad \text{--- (2)}$$

$$V_A x + H(x \tan\alpha - y) = \sum M_{xp} \quad \begin{matrix} \leftarrow \\ \text{(moments due to forces to the left of } x\text{)} \end{matrix}$$

Substituting, V_A from (1)

$$V_A = \frac{\sum M_{BP}}{L} - H \tan\alpha$$

$$\therefore \left(\frac{\sum M_{BP}}{L} - H \tan\alpha \right) x + H x \tan\alpha - Hy = \sum M_{xp}$$

$$\frac{\sum M_{BP} x}{L} - H x \tan\alpha + H x \tan\alpha - Hy = \sum M_{xp}$$

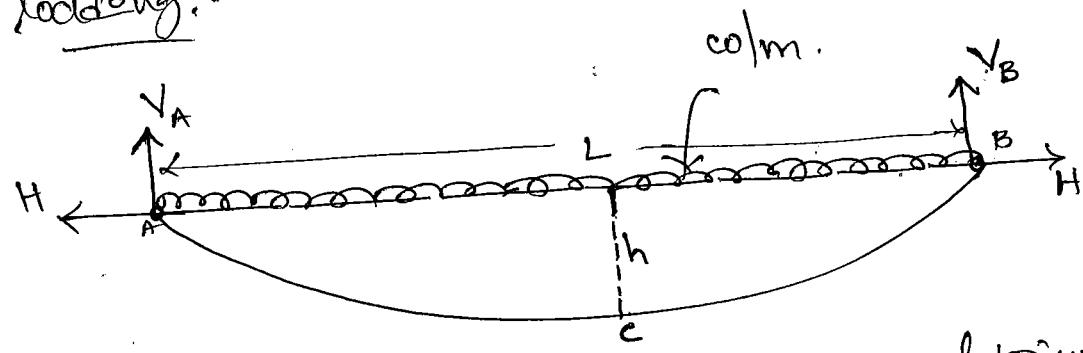
$$Hy = \frac{\sum M_{BP}(x)}{L} - \sum M_{xp}$$

$$\text{from SSB, } R_A = \frac{\sum M_{BP}}{L}$$

$$\text{BM at a distance } x \text{ for the beam} = \frac{\sum M_{BP} x}{L} - \sum M_{xp}$$

$$\therefore Hy = \text{Moment at } x \text{ for SSB}$$

Application of cable theorem for distributed loading:-



$$\Rightarrow V_A = V_B = \frac{\omega L}{2} \quad \text{from vertical equilibrium.}$$

from general cable theorem,

$$HY = \left(\frac{x}{L}\right) \sum M_{BP} - \sum M_{xp}$$

$$= \frac{x}{L} \left(\frac{\omega L^2}{2} \right) - \frac{\omega x^2}{2} = \frac{\omega x}{2} (L-x)$$

$$\therefore HY = \frac{\omega x}{2} (L-x) \quad \text{--- (1)}$$

applying general cable theorem at point 'C'.

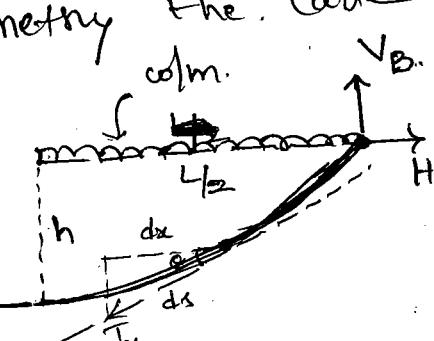
$$x = \frac{L}{2}; \quad y = h \Rightarrow H \cdot h = \frac{\omega L}{4} \left(L - \frac{L}{2} \right)$$

$$H \cdot h = \frac{\omega L}{4} \times \frac{L}{2} = \frac{\omega L^2}{8}$$

$$\therefore H \cdot h = \frac{\omega L^2}{8} \quad \text{--- (2)}$$

at point 'C', due to symmetry the cable tension is horizontal.

$$\text{from eqn (2), } H = \frac{\omega L^2}{8h}$$



$$\text{from eqn (1), } y = \frac{\omega x}{2H} (L-x)$$

To define general shape of cable in terms of mid-span def 'h',

$$y = \frac{\cos x}{2H} (L-x) \quad (\because \text{substituting for } H)$$

$$y = \frac{4hx^2}{2 \times 4L^2} (L-x) \quad \boxed{\therefore y = \frac{4hx^2}{L^2} (L-x)}$$

$$\therefore y = \frac{4hx}{L^2} (L-x) \quad (\text{eq1})$$

Let T be the axial tension in cable at a distance x . This, axial tension acts along the tangent of the cable geometry. Let us measure the length of cable 's'.

$$\begin{aligned} (\text{coso}) T = H \Rightarrow T &= \frac{H}{\cos \theta} = \frac{H}{\left(\frac{dy}{dx}\right)} = H \cdot \left(\frac{dx}{dy}\right) \\ &= H \cdot \sqrt{dx^2 + dy^2} = H \cdot \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \rightarrow ① \end{aligned}$$

$$\text{but, } \frac{dy}{dx} = \frac{d \left[\frac{4hx}{L^2} (L-x) \right]}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{4h(L-2x)}{L^2}} \quad \begin{matrix} \text{Substituting this} \\ \text{in eqn ①} \end{matrix}$$

$$T = H \sqrt{1 + \frac{16h^2}{L^4} (L-2x)^2}$$

* when $x=0$ and $x=L$, maximum Tension occurs also slope is maximum.

* At mid span, minimum Tension occurs.

To obtain total length of cable 'S'.

$$S = \int_0^L ds = \int_0^L \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad \left[y = \frac{4hx^2}{L^2} \right]$$

$$= 2 \int_0^{L/2} \left[1 + \left(\frac{8hx}{L^2} \right)^2 \right]^{1/2} dx = 2 \int_0^{L/2} \left[1 + \frac{64h^2x^2}{L^4} \right]^{1/2} dx.$$

Binomial theorem.

$$= 2 \int_0^{L/2} \left[1 + \frac{1}{2} \left(\frac{64h^2x^2}{L^4} \right) + \dots \right] dx$$

$$= 2 \left[x + \frac{32h^2}{L^4} \left(\frac{x^3}{3} \right) \right]_0^{L/2}$$

$$= 2 \left[\frac{L}{2} + \frac{32h^2}{L^4} \times \frac{1}{3} \times \frac{L^3}{3} \right]$$

$$\boxed{S = L + \frac{8h^2}{3L}}$$

* The shape of a flexible cable supported at two ends and hanging only under its self-weight is known as a CATENARY.

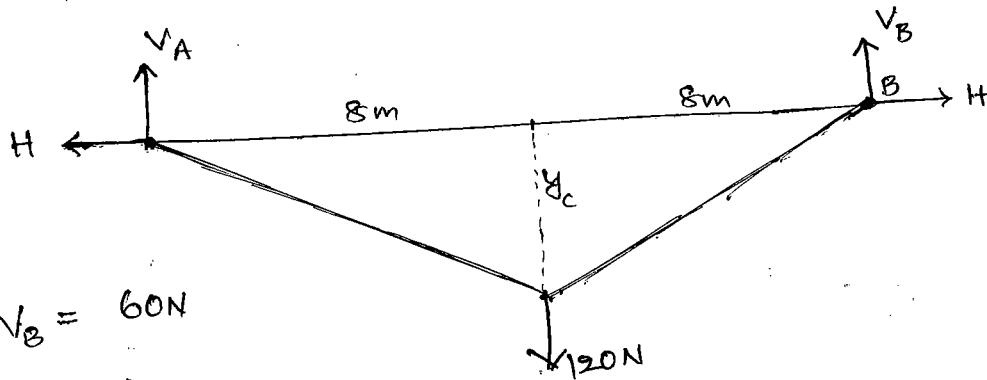
* The self wt of the cable is uniform along its curved length and not along its span. The vsl wtm that is considered for obtaining equation 1 is uniform along the span and (Galileo) not along the curved shape.

The equation of catenary is $y = a \cosh\left(\frac{x}{a}\right)$
 (Leibniz) $= \frac{a}{2} (e^{\frac{x}{a}} + e^{-\frac{x}{a}})$

- ① A light cable 18m long is supported at two ends at the same level. The supports are 16m apart. The cable supports 120N load dividing the distance into two equal parts. Find the shape of the cable and tension in cable?

SA-II

III-5



$$V_A = V_B = 60\text{N}$$

$$\left(\sqrt{8^2 + y_c^2} \right) \times 2 = 18$$

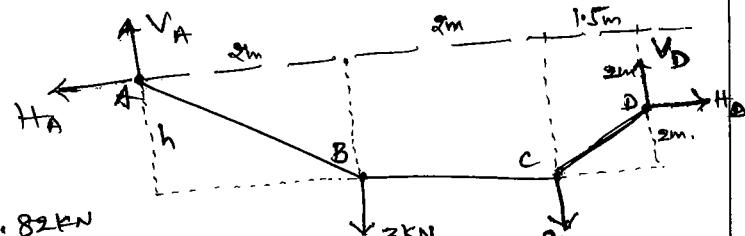
$$y_c = 4.123\text{m.}$$

$$\text{At } M_C = 0; \quad H \times 4.123 = 60 \times 8$$

$$H = 116.42\text{KN.}$$

$$T = \sqrt{H^2 + V_A^2} = \sqrt{(60)^2 + (116.42)^2} = 130.97\text{N}$$

- Q2) Determine the tension in each segment of the cable shown below. Also calculate the dimension 'h'?



$$\text{Ans} T_{CD} = 6.79\text{KN}$$

$$\text{Ans} T_{BC} = 32.3^\circ; \quad T_{BC} = 4.82\text{KN}$$

$$\text{Ans} T_{AB} = 53.8^\circ; \quad T_{AB} = 6.9\text{KN}$$

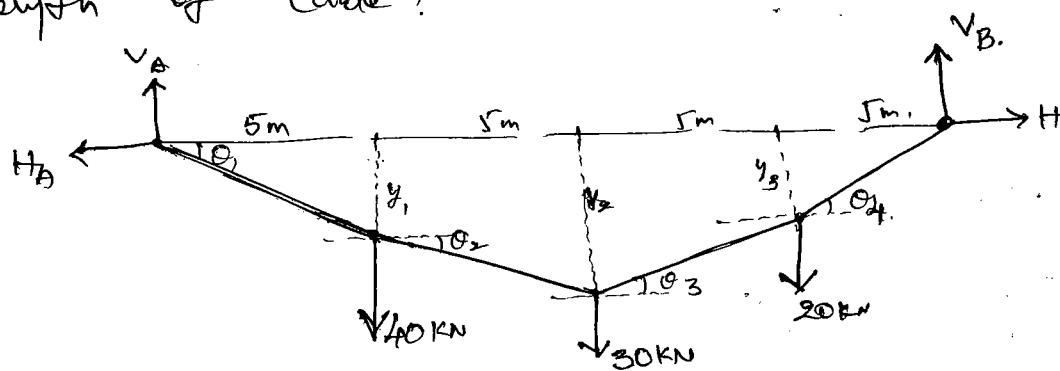
$$h = 8.74\text{m.}$$

By taking $\sum M_A = 0$, we get T_{CD} . (from cable FBD)

By taking $\sum M_A = 0$, we get T_{BC}, T_{AB} .

By taking FBD of points C, B we get T_{BC}, T_{AB} .

prob. A light cable is supported at two points 20m apart which are at same level. cable supports three concentrated loads as shown below. The deflection at center point is found to be 0.8m. Det the tension in the different segments and total length of cable?



$$\text{Ans} \quad V_A = 50 \text{ kN} : V_B = 40 \text{ kN}$$

$$y_1 = 0.8 \text{ m} ; \quad y_2 = 0.96 \text{ m} ; \quad y_3 = 0.64 \text{ m}$$

$$\theta_1 = 9.09^\circ ; \quad \theta_2 = 1.833^\circ ; \quad \theta_3 = 3.662^\circ ; \quad \theta_4 = 7.29^\circ$$

$$T_1 = 316.49 \text{ KN} ; \quad T_2 = 312.67 \text{ KN} ; \quad T_3 = 313.16 \text{ KN}$$

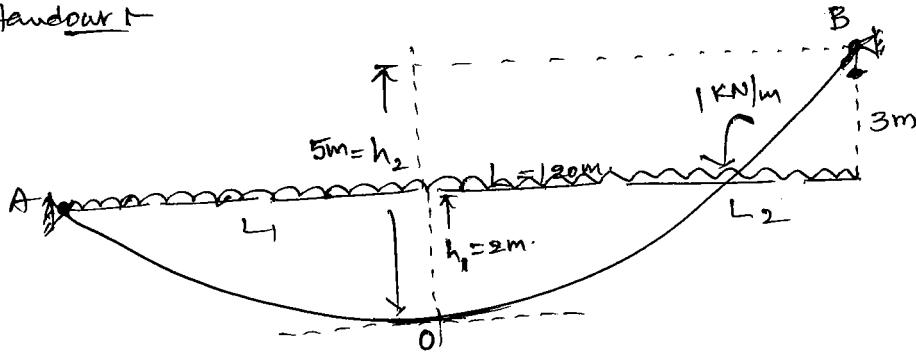
$$S = 20.11 \text{ fm}$$

$$T_4 = 315.07 \text{ KN}$$

Cables with ends at different levels:-

SA-II
III - 6

prob 6 from Handout



Considering Origin at 'O'. $y = \frac{4h_1^2}{L^2}$

$$L_1 = \frac{L \sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} ; L_2 = \frac{L \sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}$$

$$L_1 = 46.5 \text{ m} ; L_2 = 73.5 \text{ m.}$$

$$\sum M_A = 0 \quad V_B \times 120 - H \times 3 - 120 \times 60 = 0$$

$BM_O = 0$ (Taking OB Segment)

$$V_B \times 73.5 - H \times 5 - 1 \times \frac{73.5}{2}^2 = 0$$

$$V_B = 73.5 \text{ KN} \quad H = 542.4 \text{ KN.}$$

$$V_A = 46.5 \text{ KN}$$

$$\therefore T_A = \sqrt{V_A^2 + H^2} = 545.4 \text{ KN}$$

$$T_B = \sqrt{V_B^2 + H^2} = 545.4 \text{ KN.}$$

$\therefore T_{\max} = 545.4 \text{ KN}$

Temp. Effect on cables:-

$$S = L + \frac{8h^2}{3L}$$

$$\frac{dS}{dt} = \frac{16h dh}{3L}$$

$$dS = \frac{16h}{3L} dh$$

$$SS = S \alpha t = \alpha t \left(L + \frac{8h^2}{3L} \right) = L \alpha t + C$$

$$SS = L \alpha t$$

$$\therefore 8h \times \frac{16h}{3L} = L \alpha t$$

$$8h = \frac{3L^2 \alpha t}{16h} = \frac{3L}{16h} (L \alpha t)$$

Taking moment about lower pt:-

$$H \cdot h = M_c \Rightarrow H = \frac{M_c}{h}$$

$$\frac{dH}{dh} = -\frac{M_c}{h^2} = -\left(\frac{M_c}{h}\right) \times \frac{1}{h}$$

$$\frac{dH}{dh} = -\frac{H}{h}$$

$$\frac{dH}{H} = -\frac{8h}{h}$$

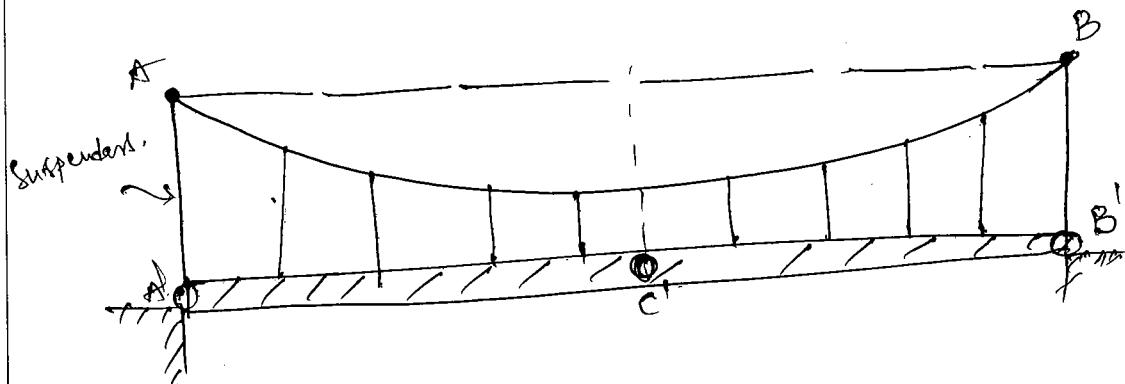
$$\frac{dH}{H} = -\frac{3L(L \alpha t)}{16h^2}$$

$$* \quad dH = -\frac{3L(L \alpha t)}{16h^2} [H] \quad \begin{cases} H \text{ decreases} \\ \text{with increase in Temp} \end{cases}$$

Suspension Bridge with one stay-cable girder

SA-II

III-7



prob:

A 3H stiffening girder of a suspension bridge of span 100m is subjected to two point loads of 200 KN and 300 KN at a distance of 25m & 50m from left. Find SF and BM for the girder at a distance 30m from left. The supporting cable has a central dip of 10m. Find also max tension and its slope in the cable?

Sol:

Taking FBD of cable & S.G. \Rightarrow

$$\sum M_A = 0 \Rightarrow R_{B'} \times 100 - 200 \times 25 - 300 \times 50 = 0 \\ R_{B'} = 200 \text{ KN.}$$

FBD of S.G.: - $(B'C')$

$$M_{C'} = 0 \Rightarrow R_B \times \frac{l}{2} - \frac{\omega l^2}{8} = 0$$

$$R_B \times \frac{l}{2} = \frac{\omega l^2}{84} \Rightarrow 200 = \omega \times \frac{100}{4} \Rightarrow$$

$$\omega = 8 \text{ KN/m.}$$

$$R_A = 300 \text{ KN.}$$

$$SF_{\text{at } 30m} = 300 - 200 - \omega_e \left(\frac{l}{2} - x \right) = -60 \text{ KN}$$

$$BM = 300 \times 30 - 200 \times 5 - \frac{\omega_e l}{2} (l-x) = -400 \text{ KN.m.}$$

for cable:- $V_A = \frac{W_e l}{2} = \frac{8 \times 100}{2} = 400 \text{ kN}$

$$H \cdot h = \frac{W_e l^2}{8} = \frac{8 \times 100^2}{8} = 10,000$$

$$H = \frac{10,000}{10} = 1000 \text{ kN}$$

$$T_{\max} = \sqrt{1000^2 + 400^2} = 1077.03 \text{ kN}$$

$$T_{\max \text{ cos } \theta} = H \Rightarrow \cos \theta = \frac{1000}{1077.03} \Rightarrow \theta = 21.8^\circ$$

INTU

Prob:- A 3H Suspension girder bridge has a span of 200m over the supports at same level. It has a central dip of 80m. The girder carries three point loads of 15kN, 25kN and 20kN acting at 35m, 80m & 150m resp. from left end. Draw BMD?

Prob:- A Suspension Cable 100m span and 15m dip is stiffened with a three hinged girder. If a concentrated load of intensity 100kN acts at 30m from left. Determine max tension in the cable. Also det. max BM & SF?

Suspension cable with two-hinged stiffening girder:-

Girder:-

$$w_e = \frac{W}{L}$$

SA-II

III - ⑧

This is statically Indeterminate structure. However the analysis is made simple by assuming that the girder is very stiff and hence whatever is the load on the girder the stiffeners are equally stressed.

Prob:- A suspension bridge of span 80m ~~is~~ and width 6m is having two cables stiffened with two hinged girders. The central dip of cables is 8m. Dead load on the bridge is 5KN/m^2 and LL is 10KN/m^2 which covers the left half of span. Determine S.F and BM at 20m from left end. Find also the max tension in the cable?

Sol:- $l = 80\text{m}, \quad h = 8\text{m}, \quad D.L = \frac{5 \times 6}{2} = 15\text{KN/m}$
 $L.L = \frac{10 \times 6}{2} = 30\text{KN/m}$ (last half)

$$W = 15 \times 80 + 30 \times 40 = 2400\text{KN}$$

$$C.O.E = \frac{W}{l} = \frac{2400}{80} = 30\text{KN/m.}$$

$$H = \frac{w_e l^2}{8h} = \frac{30 \times 80^2}{8 \times 8} = 3000\text{KN}$$

$$V = \frac{C.O.E l}{2} = \frac{30 \times 80}{2} = 1200\text{KN.}$$

$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{3000^2 + 1200^2} = 3231.1\text{KN.}$$

Girder $R_A = \frac{15 \times 80 \times 40 + 30 \times 40 \times (80-20)}{80} = 1500\text{KN.}$

$$\text{Beam SF}_D = 1500 - 1500 - 30 \times 20 = 600\text{KN}$$

$$SF \text{ at } D^1 = 600 - \left(\frac{30x80}{2} - 30x20 \right)$$

$$= 600 - (1200 - 600) = 0$$

$$\Delta M_B = 1500 \times 20 - 15 \times 20 \times 10 - 30 \times 20 \times 10$$

$$- \left(\frac{30 \times 20 \times 80}{2} - 30 \times \frac{20 \times 20}{2} \right) = 3000 \text{ kNm}$$

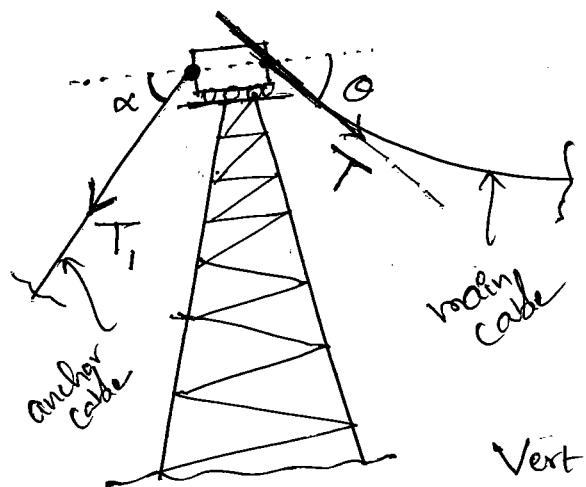
Prob: A suspension cable 100m Span and 12m drop is stiffened with a 2H girder. The girder carries DL of 10kN/m over entire span and a load of 800 kN at 40m from left. Det. max tension in cable and SF in BM ~~from~~ at 30m from left?

Anchor cable: main cable is supported by anchor cable in two ways.

SA-II
iii - 9

- (i) Saddle (ii) pulley.

(i) Saddle :- Horizontal roller on tower.



$$\sum F_x = 0 \Rightarrow$$

$$T \cos \theta = T_1 \cos \alpha \quad \text{---(1)}$$

θ, α, T → solved from cable.

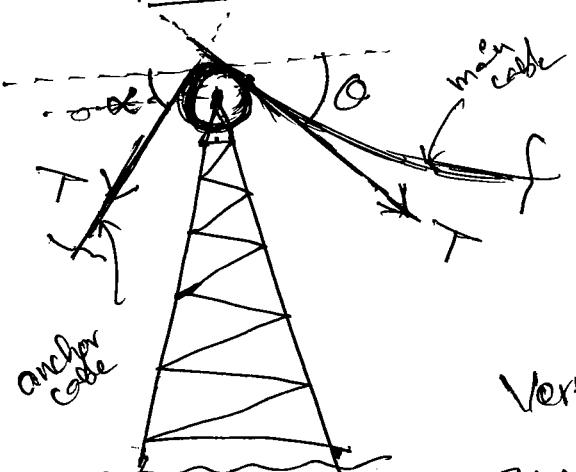
T_1 may be calculated from (1)

$$\text{Vert force} = T \sin \theta + T_1 \sin \alpha$$

Moment at base of tower = can be calculated mounted on tower.

(ii) Pulley :-

main cable T, θ, α solved from cable.



$$\begin{aligned} \text{Horz force} &= T \cos \theta - T \cos \alpha \\ &= T(\cos \theta - \cos \alpha) \end{aligned}$$

$$\text{Vert force} = T(\sin \theta + \sin \alpha)$$

BM at base of tower = can be calculated

prob
 A Bridge cable is suspended from towers 80m apart and carries a load of 30kN/m on the entire span. If the max sag is 8m. calculate maximum tension in cable if the cable is supported by saddles which are stayed (anchored) by wires inclined at 30° to horizontal. Determine forces on tower. If the same inclination of back stay passes over pulley determine forces on towers?

~~left~~ $V_A = 1200 \text{ kN}$ Pulley moves at 'C'
 $H = 3000 \text{ kN}$

$$T_{max} = \sqrt{1200^2 + 3000^2} = 3231.1 \text{ kN}$$

$$H = T_{max} \cos \theta \Rightarrow \theta = 21.8^\circ$$

Saddle L $T_1 \cos \theta = T_{max}$

$$T_1 = 3464.1 \text{ kN}$$

$$\text{Vert force} = 2932.0 \text{ kN}$$

pulley R $\text{Hor force} = 201.82 \text{ kN}$

$$\text{vert force} = 2815.48 \text{ kN}$$

Stiffness and carry over factor - Distribution factor
 Analysis of Continuous beams with and without Sinking
 portal frames - including Sway. Substitute frame of supports.
 Analyze by two cycle

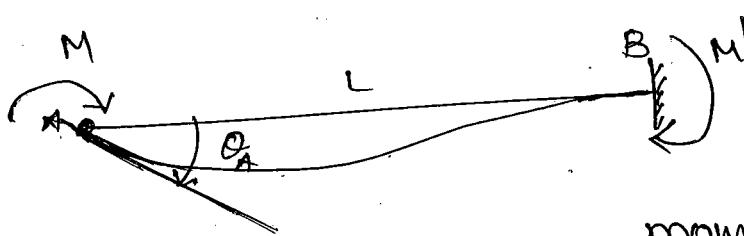
Intro:- * This method is suggested by prof Hardy Cross in 1930's

- * Solution of Simultaneous equations in SDM is replaced by an iterative distribution procedure.

$$\text{SDE are, } M_{AB} = M_{FAB} + \frac{\alpha EI}{L} \left[(\theta_A + \theta_B) - \frac{3A}{L} \right]$$

$$M_{BA} = M_{FBA} + \frac{\alpha EI}{L} \left[(\theta_B + \theta_A) - \frac{3A}{L} \right]$$

COM :- carry over moment; when a member moment is applied at one end of a member allowing rotation of that end and fixing the far end. Some moment develops at the far end also, this is called COM.



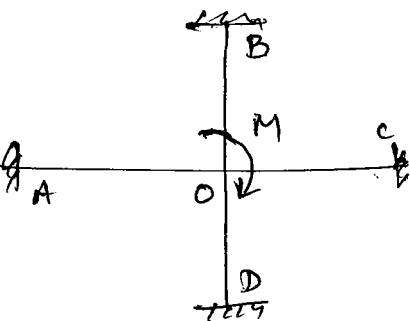
$$\text{COF} := \frac{M'}{M} = \frac{\text{Carry over moment}}{\text{Applied moment.}}$$

Stiffness :- moment required to rotate an end by unit angle.

$$K_O = \frac{M}{\theta_2}$$

D.F.: Distribution Factor. The ratio of moment shared by a member to the applied moment at the joint is D.F.

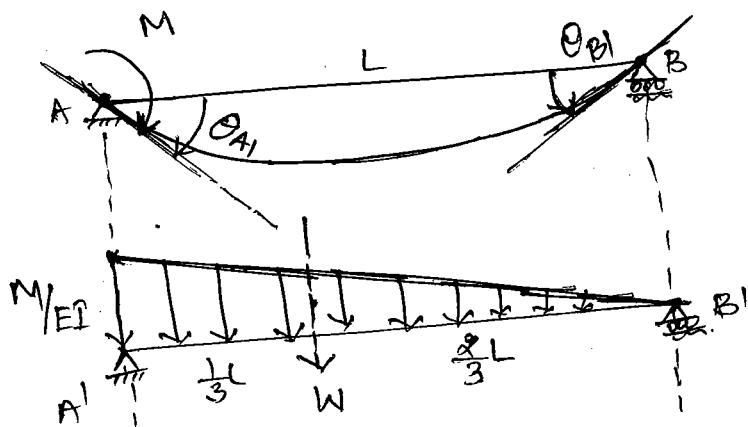
$$D.F \text{ of } OA = \frac{M_{OA}}{M}$$



Sign Convention: 1. C.W moments are +ve
2. A.C.W " -ve

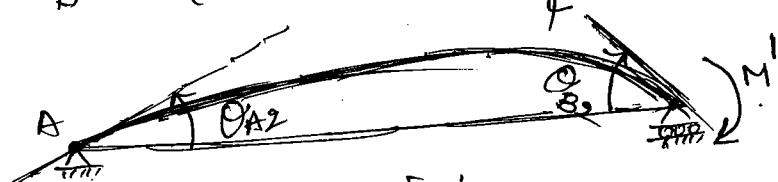
Expressions for COF, D.F., Stiffness

CBM:- Consider a SSB of span L applied by moment M at A



$$\Theta_{A1} = R_{A1} = \left(\frac{1}{2} \times L \times \frac{M}{EI} \right) \times \frac{4}{3} = \frac{ML}{3EI}$$

$$\Theta_{B1} = R_{B1} = \left(\frac{1}{2} \times L \times \frac{M}{EI} \right) \times \frac{1}{3} = \frac{ML}{6EI}$$



Applying Redundant M' at B'

$$\Theta_{A2} = \frac{M'L}{6EI} \quad \Theta_{B2} = \frac{M'L}{3EI}$$

But, end B of the beam is pinned

SA-II

hence, $\theta_{B1} = \theta_{B2} = 0$

IV - Q

$$\theta_{B1} = \theta_{B2} \Rightarrow \frac{Mk}{6EI} = \frac{M'k}{3EI}$$

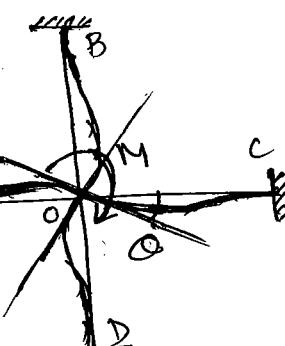
$$M' = \frac{M}{2} \Rightarrow \frac{M'}{M} = \frac{1}{2} = COF$$

$$\begin{aligned}\theta_A &= \theta_{A1} - \theta_{A2} = \frac{ML}{3EI} - \frac{M'L}{6EI} \\ &= \frac{ML}{3EI} - \left(\frac{M}{2}\right)\frac{L}{6EI} \\ &= \frac{ML}{3EI} - \frac{ML}{12EI} = \frac{ML}{EI} \left(\frac{12-3}{12 \times 3} \right) \\ &= \frac{ML}{EI} \times \frac{9/3}{4} = \frac{ML}{4EI} \\ \therefore K_o &= \frac{M}{\theta_A} = \frac{M}{\frac{ML}{4EI}} = \boxed{\frac{4EI}{L} = K_o}\end{aligned}$$

Consider a joint consisting of four members -

$$M = M_1 + M_2 + M_3 + M_4$$

$$\theta = \frac{M}{K_i} = \frac{M_2}{K_2} = \frac{M_3}{K_3} = \frac{M_4}{K_4}$$



$$\theta = \frac{M}{K} = \frac{M_1 + M_2 + M_3 + M_4}{K_1 + K_2 + K_3 + K_4}$$

$$= \frac{M}{\sum_{i=1}^4 K_i}$$

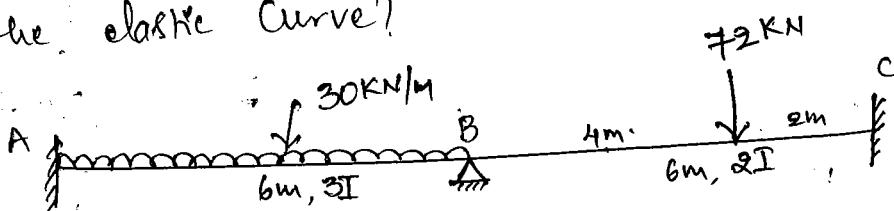
$$M_i = K_i \theta = K_i \left(\frac{M}{\sum K_i} \right) \Rightarrow DF = \frac{M_i}{M} = \frac{K_i}{\sum K_i}$$

procedure:-

1. Assume all ends are fixed and find fixed end moments.
2. find D.F. for all members at a joint.
3. Balance a Jt by distributing balancing moment to various members meeting at the joint proportional to their D.F. Apply same procedure to all joints.
4. carry over half the distributed moment to far ends. this upsets balance of Jnt.
5. Repeat step 3 & 4 till distributing moments are negligible.
6. Sum all moments at a particular end of a member to get final moment.

Note :- fixed end is a self balancing Jnt.

prob:- Analyse the continuous beam shown by MDM and draw BMD & SFD. Also draw the elastic curve?



sol:- FEM's :- $M_{FAB} = -\frac{Pab^2}{12} = -\frac{30 \times 6^2}{12} = -90 \text{ kNm}$

$$M_{FBA} = 90 \text{ kNm}$$

$$M_{FBC} = -\frac{Pab^2}{12} = -\frac{72 \times 4 \times 2^2}{6^2} = -32 \text{ kNm}$$

$$M_{FCB} = \frac{Pba^2}{12} = \frac{72 \times 2 \times 4^2}{6^2} = 64 \text{ kNm}$$

D.F.

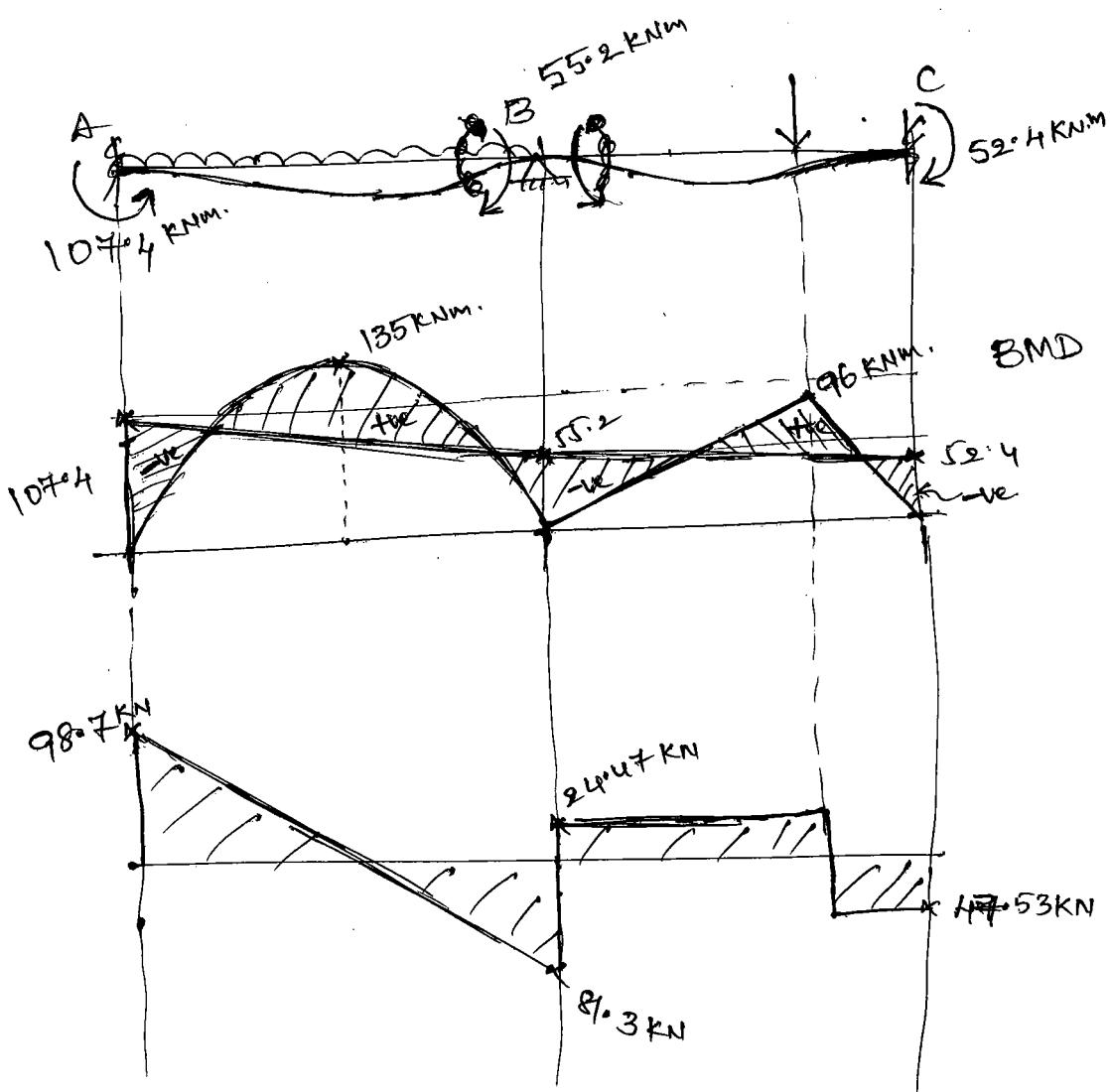
SA-II

IV - ③

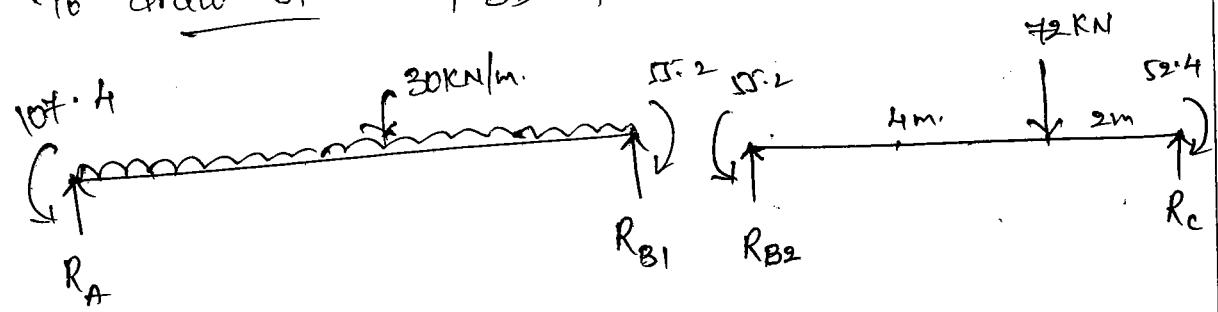
Jt	Members	K	ΣK	DF
B	BA	$\frac{4E3I}{6} = 2EI$	$3.33EI$	0.6
	BC	$\frac{4E(3I)}{6} = 1.33EI$		0.4

Distribution Pattern

	A	B	C
D.F.		0.6 0.4	
F.E.Ms	-90	90 -32	64
Bal.		-34.8 -23.2	
COM.	-17.4		-11.6
Final End Moments	-107.4	55.2 -55.2	52.4



To draw SFD - FBD of each exam.



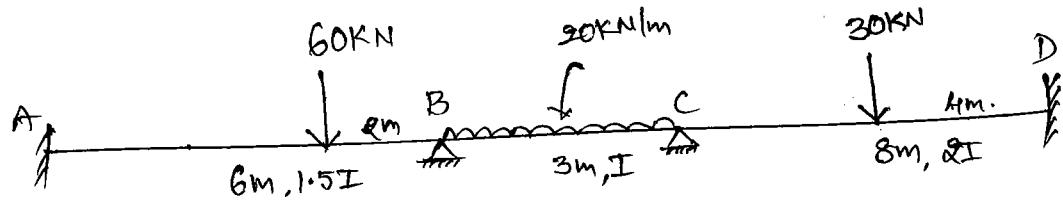
$$R_{B1} = 81.3 \text{ kN}, R_A = 98.7 \text{ kN}$$

$$R_{B2} = 24.47 \text{ kN}, R_C = 47.53 \text{ kN}$$

- C.H. prob
- ① Using MDM analyze the two Span Continuous beam. The moment of inertia of AB = I while that of BC = αI . The ends A & C are fixed. Sketch the BMD & S.F.D. Span AB carries a concentric load of 36 kN with a span of 6m and span BC carries a UDL of 20 kN/m over a span of 8m?

prob: Analyse the continuous beam shown below and draw BMD?

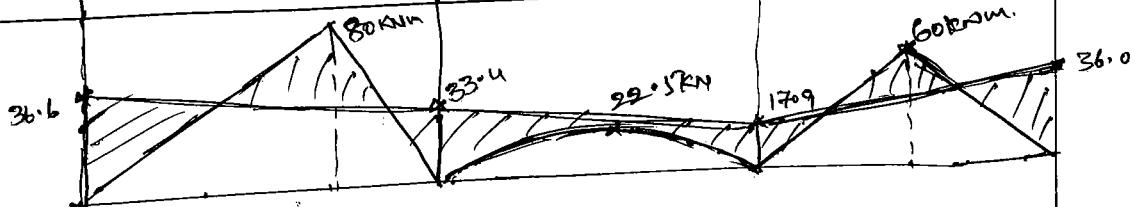
SA-II
IV-④



Sol:-

$M_{FAB} = -26.67$	$M_{FBC} = -15$	$M_{FCD} = -30 = \frac{PL}{8}$
$M_{FBA} = 53.33$	$M_{FCB} = 15$	$M_{FDC} = 30 = \frac{PL}{8}$
$DF_{BA} = 0.43$	$DF_{CB} = 0.57$	
$DF_{BC} = 0.57$	$DF_{CD} = 0.43$	

M.D.	A	B	C	D
D.F		0.43	0.57	0.43
FEM	-26.67	53.33	-15	15
Bal		-16.48	-21.85	8.45
COM	-8.24	4.28	-10.93	3.23
Bal		-1.84	-2.44	6.23
COM	-0.92	3.12	-1.22	2.35
Bal		-1.34	-1.78	0.7
COM	-0.67	0.35	-0.89	0.27
Bal		-0.15	-0.2	0.51
COM	-0.075	0.25	-0.1	0.16
Bal		-0.11	-0.14	0.05
Final end moments	-36.6	33.4	-33.4	17.9
				36



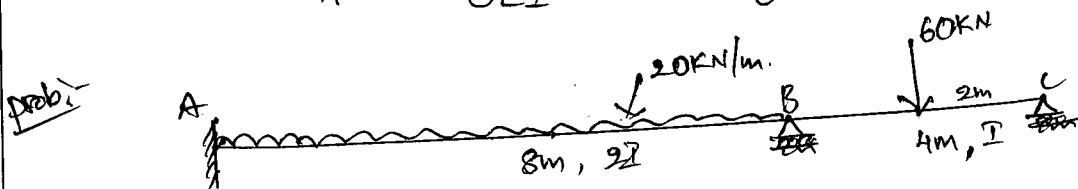
BMD

CONTINUOUS beam with simple end:-



$$M = 0 \Rightarrow \text{COM} = 0 \quad \text{COF} = 0$$

$$R_A = \frac{ML}{3EI} \Rightarrow \frac{M}{R} = \frac{3EI}{L} = K_A$$



$$M_{FAB} = -106.67 \quad M_{FCB} = -30$$

$$M_{FBA} = 106.67 \quad M_{FCB} = 30$$

D.F.L

~~D.F.M.~~

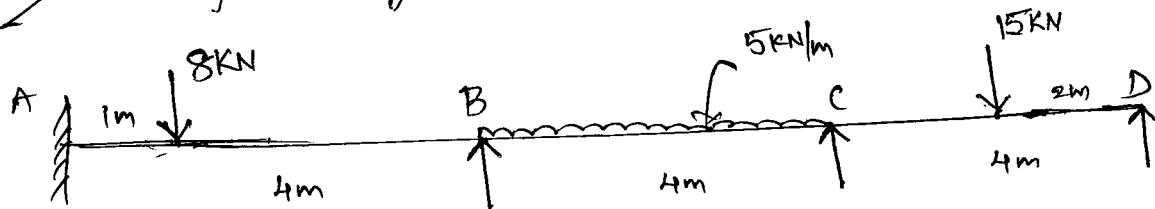
IT	mems	K _A	ΣK	D.F
B	R _A R _C	$\frac{3EI}{L} = 0.75EI$	1.75EI	0.57 0.43

M.D :-

	A	B	C
DF		0.57	0.43
FEM	-106.67	106.67	-30
Bot			-30
COM		-15	
Bot		-35.15	-26.12
COM	-17.6		0
Final	-124.57	71.52	-71.52
			0

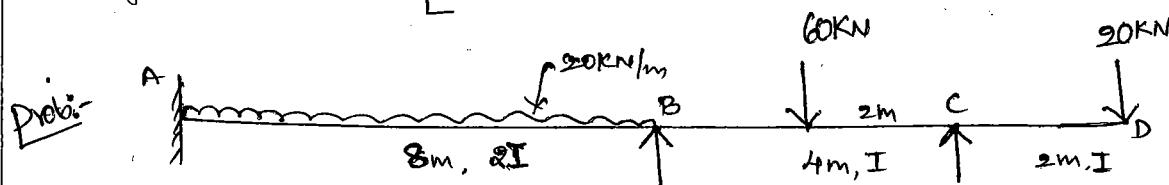
May-2016 Draw FBD for continuous beam shown below
by using MDM?.

S-A-II
N-5



continuous beams with Over hang:-

- * Stiffness of overhanging portion is taken as zero (D.F=0). Since moment in this portion do not depend on loading in other portions. Support C is considered similar to simply supported end. Hence stiffness of BC is $\frac{3EI}{L}$ and no carry over from B to C.



$$\text{Ans:-} \quad M_{FAB} = -106.67 \quad M_{FBC} = -30 \quad M_{FCD} = -40$$

$$M_{FBA} = 106.67 \quad M_{FCB} = 30$$

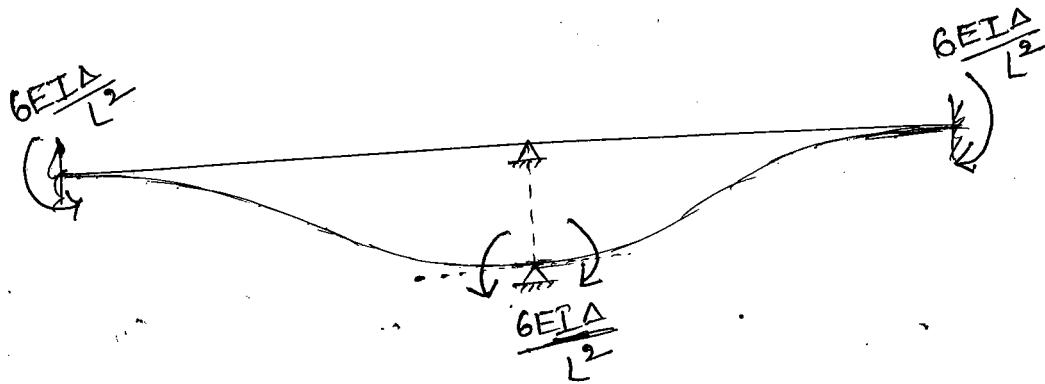
D.F:-

JE	memb	K	St	DF
B	BA	$\frac{HEI}{L} = EI$	$1.75EI$	0.57
	BC	$\frac{3EI}{L} = 0.75EI$		
C	CB	$\frac{4EI}{L} = EI$	EI	1.0
	CD	0		0

	A	B	C overhang D		
D.F.		0.57	0.43	1.0	0
FEM	-106.67	106.67	-30	30	-40
Bal.				10	0
COM			5		
Bal		-40.92	-30.75		
COM		-20.46		0	
fixed E.M.	-127.13	65.75	-65.75	40	-40

Beams with Sinking of Supports:-

- * fixed end moments due to Settlement of supports are added to fixed end moments due to loads. Rest of analysis is same.



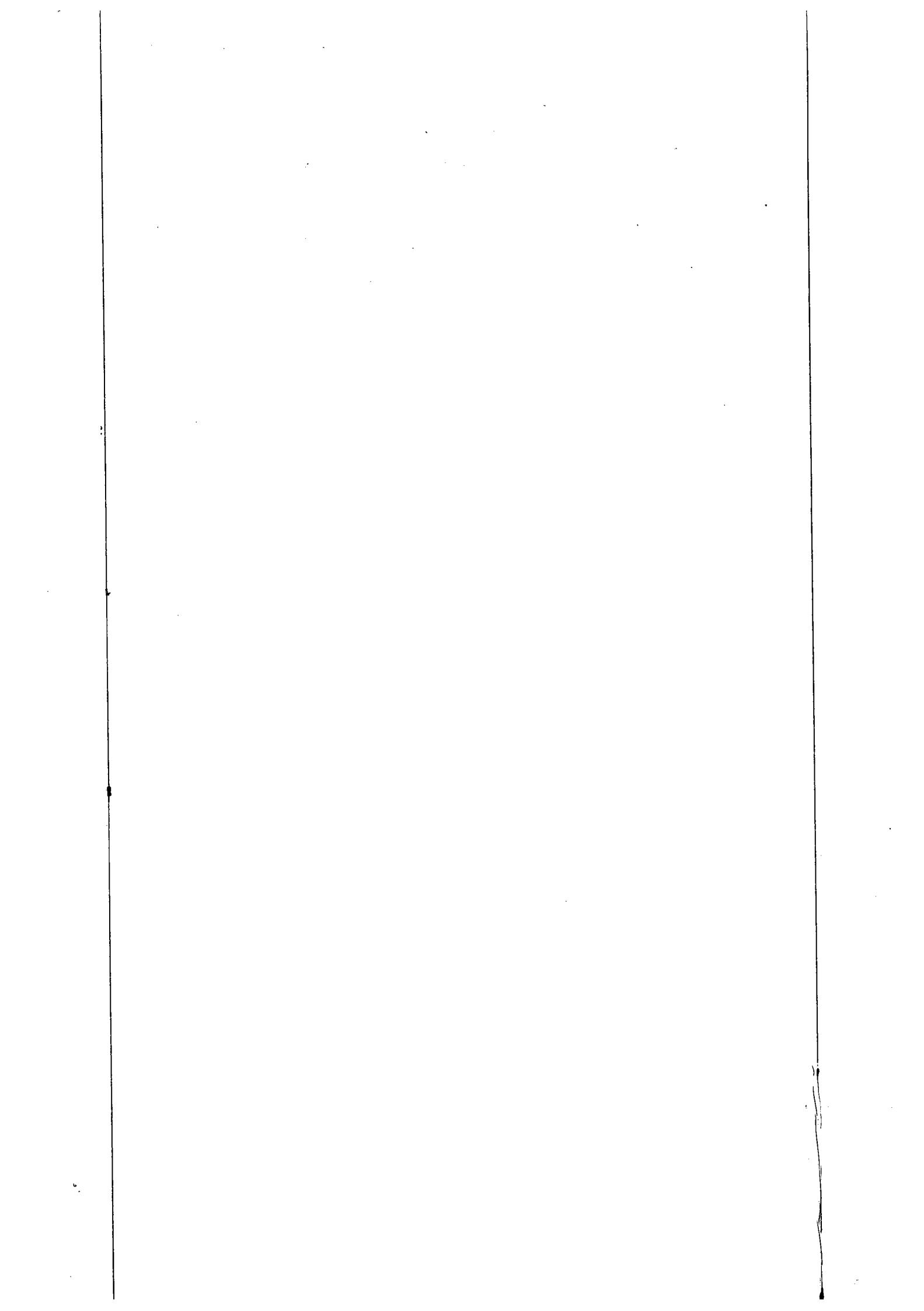
RHSS \rightarrow -ve moments

LHSS \rightarrow +ve "

prob: A fixed beam of span 6m carries a UDL of 18 kN/m. If the right support sinks by 6.5mm, find fixing end moment of the supports. Draw SFD & BMD. Take $E = 200 \text{ kN/mm}^2$, $I = 5 \times 10^7 \text{ mm}^4$. Analyse by MDM?

May/Jun
2015 A continuous beam ~~ABCD~~ is simply supported at A, B and C and is fixed at D. The spans AB, BC and CD are 3m, 4m and 6m long. The beam carries point load of 15kN on AB at 2m from A. A pt load of 25kN at middle of BC and a pt load of 8kN at middle of CD. If $I_{AB} = I_{BC} : I_{CD} = 1:1:2$ find support moments and reactions using MDM?

SA-II
IV-6



frames with sway:-

SA-II
IV - 7

- (i) Assume the sway in the frame is prevented by giving external support at beam level. carryout analysis which is called non-sway analysis.
 - (ii) Considering FBD of columns, find horz forces developed at supports, then considering horizontal equilibrium of entire system at addition beam levels find force 'S' developed at support assumed at.
 - (iii) Assuming arbitrary end moments due to sway carryout analysis to find sway force 'S'.
 - (iv) Find sway correction factor $K = \frac{S}{S'}$
 - (v) Col. find moments by the eqn.
$$\text{Final End moment} = \text{non-sway moments} + K \times \text{sway moments.}$$
- S → Sway force
S' → Arbitrary Sway force

prob 5 Analyse the portal frame by using k-m method.

Non-Sway Analysis

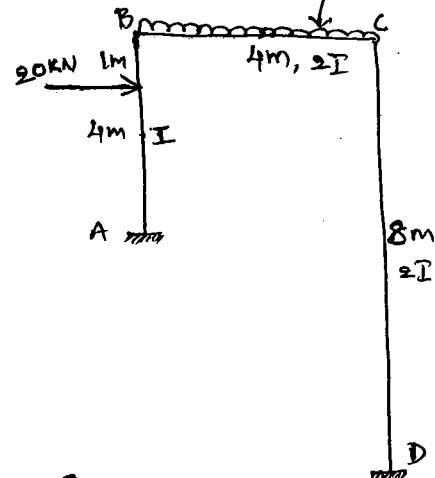
Step 1:
Assuming a Support at C.

$$M_{FAB} = -\frac{20 \times 3 \times 1^2}{4^2} = -3.75 \text{ KNm}$$

$$M_{FBA} = \frac{20 \times 3 \times 1^2}{4^2} = 11.25 \text{ KNm}$$

$$M_{FBC} = -\frac{30 \times 4^2}{12} = -40 \text{ KNm.}$$

$$M_{FCB} = +40 \text{ KNm.} ; M_{FCD} = M_{FDx} = 0$$



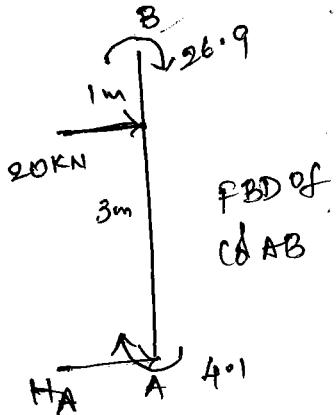
D.F

Step 2:

J.F	Memb	K _i	EK _i	D.F
B	BA	$\frac{4EI}{4} = EI$	$3EI$	0.33
	BC	$\frac{4EI(2)}{4} = 2EI$		0.67
C	CB	$\frac{4EI(3)}{4} = 2EI$	$3EI$	0.67
	CD	$\frac{4EI}{8} = EI$		0.33

Step 3, 4, 5:

D.F	A	B	C	D
FEM	-3.75	11.25	0.67	0.33
BA1	9.5	-40	40	0
COM	4.75	-13.4	9.63	-6.6
BA1	4.42	9.0	-6.45	-3.2
COM	2.21	-3.225	4.5	-1.6
BA1	1.1	2.9	-3.02	-1.5
COM	0.55	-1.51	1.1	-0.75
BA1	0.5	1.0	-0.74	-0.36
COM	0.25	-0.37	0.5	-0.18
BA1	0.12	0.25	-0.165	-0.335
COM	0.06	-0.0825	0.125	-0.1675
Fixed End Moments	4.1	26.9	-26.9	18.6
				-9.3



FBD of
CD AB

$$\sum M_B = 0 \quad 4 \cdot 1 + 26.9 - 20 \times 1 - H_A \times 4 = 0$$

$$H_A = \frac{11}{4} = 2.75 \text{ kN}$$

18.6

$$\sum M_C = 0 \quad 9 \cdot 3 + 18.6 + H_D \times 8 = 0$$

$$H_D = -3.5 \text{ kN}$$

H_D
9.3

$$\sum F_x = 0 \quad H_A + H_D + 20 - S = 0$$

$$2.75 - 3.5 + 20 - S = 0$$

$$S = 19.25 \text{ kN}$$

Sway Analysis :- Let M_{FI} & M_{F2} be fixed end moments in AB & CD columns for a sway.

$$= \frac{I^2}{4^2} \times \frac{8^2}{2^2} = \frac{8 \times 8^2}{4 \times 4 \times 2} = 2$$

$$\frac{M_{FI}}{M_{F2}} = \left(\frac{6EI_1\Delta}{L_1^2} \right) \left(\frac{6EI_2\Delta}{L_2^2} \right)$$

$$\therefore M_{FI} = -100 \quad M_{F2} = -50$$

$$\therefore \text{Let } M_{FI} = -100 \quad M_{F2} = -50 \quad M_{FCD} = M_{FDC} = -50$$

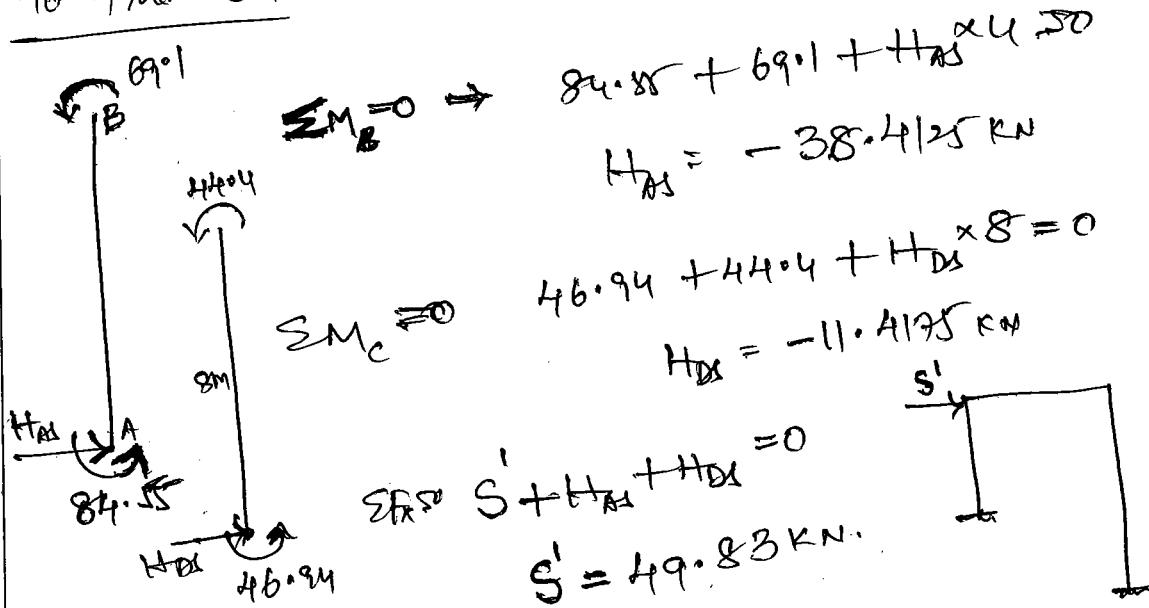
$$\therefore M_{FBA} = M_{FAB} = -100$$

M.D for Arbitrary Sway

D.F	A	B	C	D
FEMS	-100	-100	0	-50
B.M		33	67	33.5
Com	16.5	16.75	33.5	8.95
B.M		-5.53	-11.22	-8.45
Com	-2.75	-11.22	-8.45	-5.53
B.M		3.7	7.52	1.85
Com	1.85	1.88	3.76	0.925
B.M		-0.62	-1.26	-0.62
Com	-0.31	-1.26	-0.63	-0.62

COM	-0.31	-1.26	-0.63	-0.62
Bat		0.42	0.84	0.42 0.91
COM	0.21	0.21	0.42	0.11
Bat		-0.07	-0.14	-0.28 -0.14
COM	-0.035	-0.14	-0.07	-0.07
Find End moments	-84.55	-69.1	69.1	44.4 -44.4 -46.94

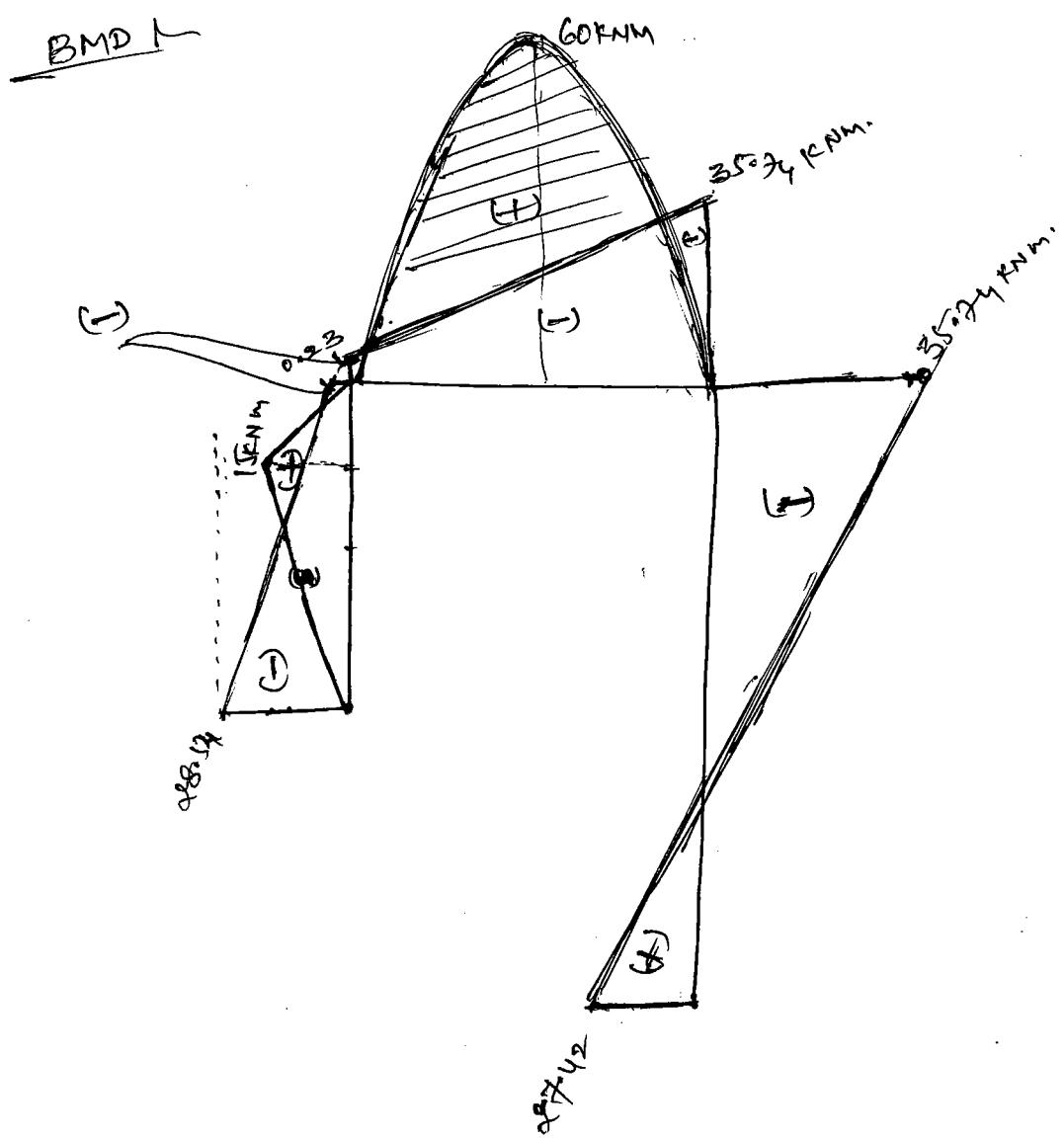
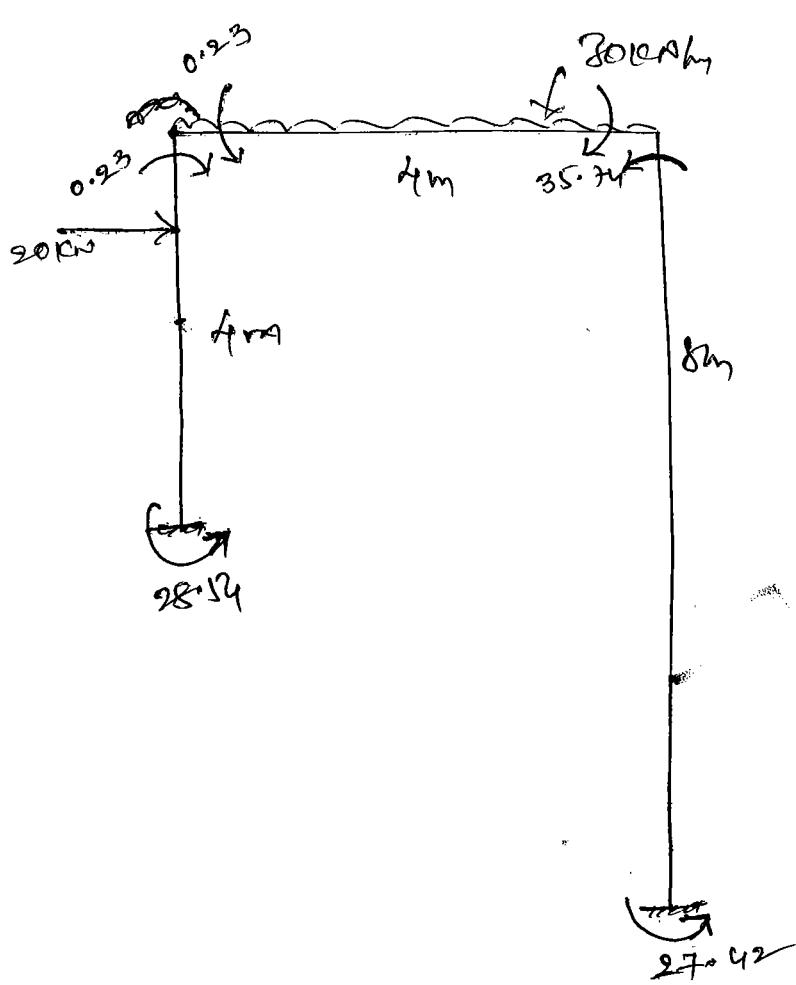
To find S' :

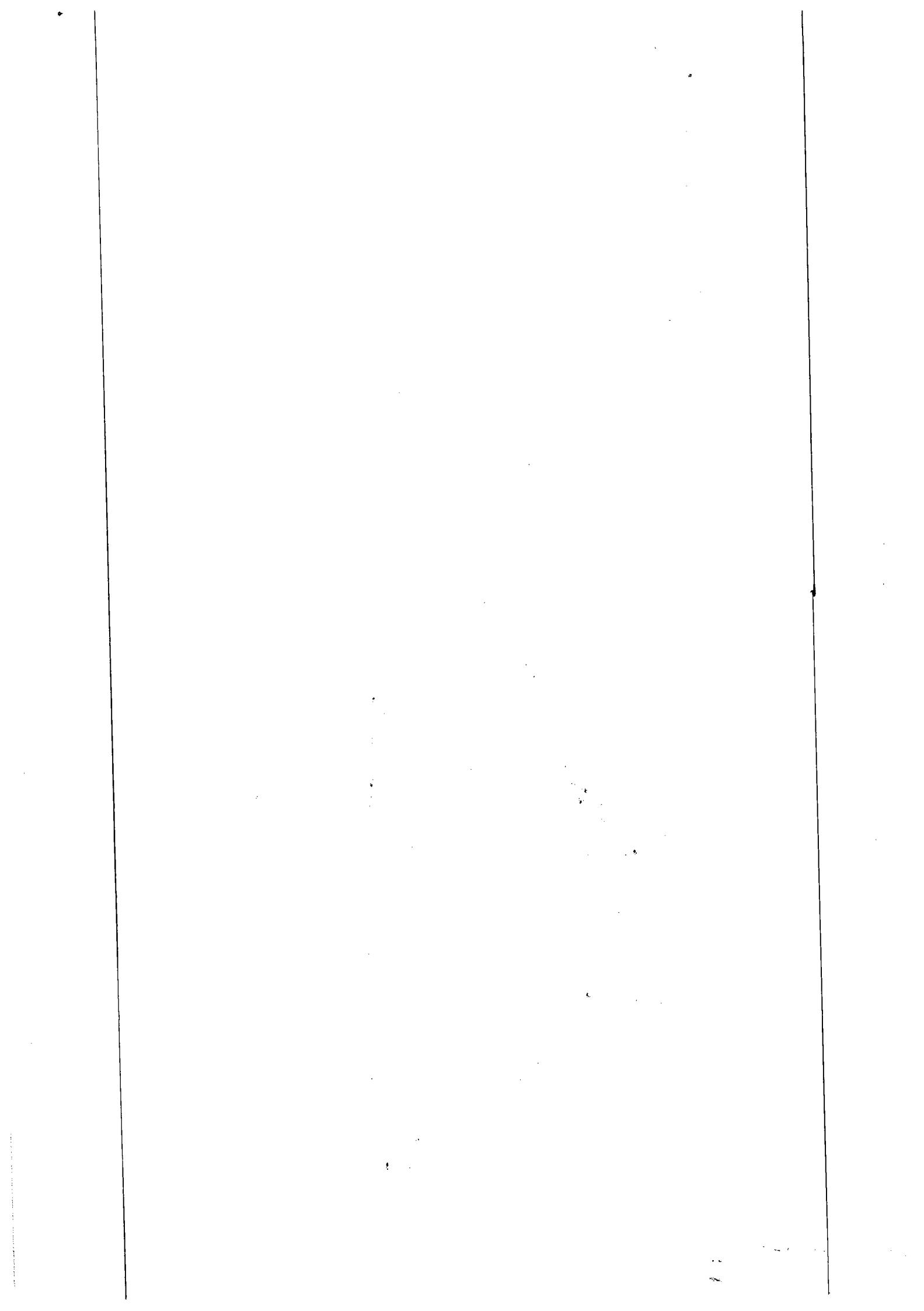


$$\therefore K = \frac{S}{S'} = \frac{19.25}{49.83} = 0.386$$

Find End moments

	A	B	C	D	
Non-Sway	40.1	26.9	-26.9	18.6	-18.6
Arbitrary Sway	-84.55	-69.1	69.1	44.4	-44.4 -46.94
Corrected Sway moments	-32.64	-26.67	26.67	17.14	-17.14 -18.12
Final (K3) End moments.	-28.54	0.23	-0.23	35.74	-35.74 -27.42





Analysis of continuous beams - including settlement of supports; Single bay portal frames with and without Side Sway.

Introduction:- * GASPER KANI, A german Engineer, developed another distribution procedure based on slope deflection equations.

- * This method is very useful for the analysis of multi storey frames.
- * The greatest advantage of this method is, even if a mistake is committed in distribution in one of the cycles, it converges finally to the correct answer.
- * Even today, many practicing engineers who are not familiar with computer methods, use Kani's method for the analysis of 3 to 4 storey building frames.

Sign Conventions: 1. clockwise end moments are positive
2. clockwise rotations are positive.

Let member AB shown below be an intermediate member of a beam/frame, which has no relative displacement at the ends.

→ Let M_{AB} and M_{BA} are final end moments

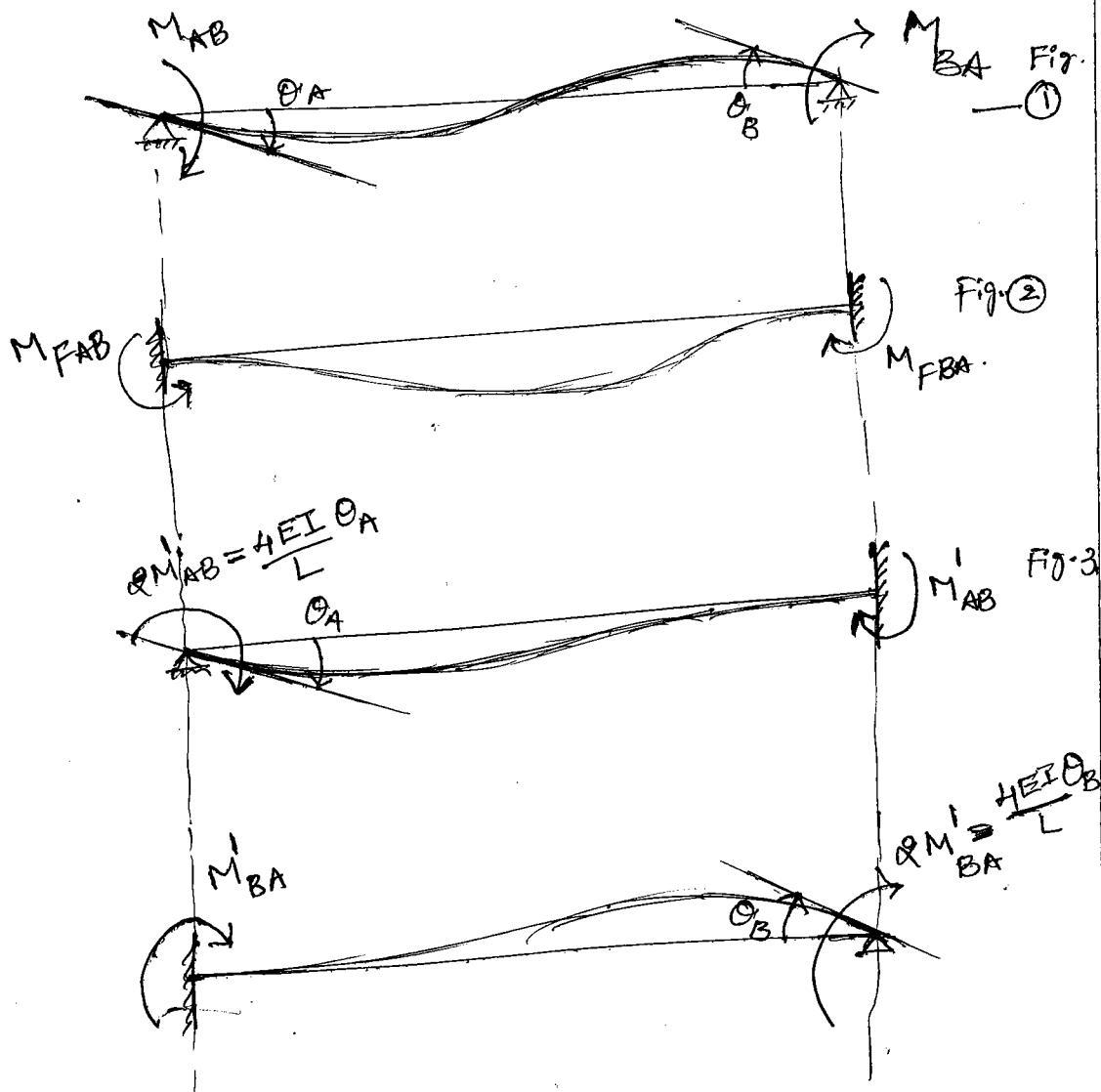
M_{AB} may consist of:

(i) Fixed end moments ($\theta_A = \theta_B = 0$)

(ii) moment due to rotation of end A only.



A " " B "



$$\therefore M_{AB} = M_{FAB} + \alpha M'_{AB} + M'_{BA}$$

$$M_{BA} = M_{FBA} + M'_{AB} + \alpha M'_{BA}$$

M'_{AB}, M'_{BA} are called rotation contributions.

\therefore Fixed end moment = fixed end moment + α (Rotation contribution of near end) + Rotation contribution of far end

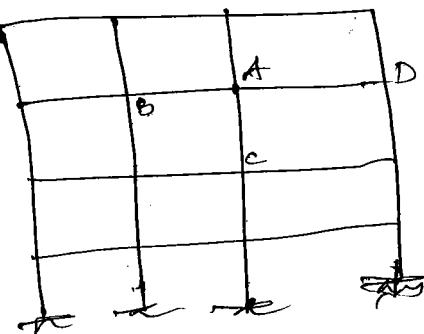
Now, consider moments at point A shown in the frame E

$$M_{AB} = M_{FAB} + \alpha M'_{AB} + M'_{BA}$$

$$M_{AC} = M_{FAC} + \alpha M'_{AC} + M'_{CA}$$

$$M_{AD} = M_{FAD} + \alpha M'_{AD} + M'_{DA}$$

$$M_{AE} = M_{FAE} + \alpha M'_{AE} + M'_{EA}$$



$$\text{at A: } \sum M_{AB} = \sum M_{FAB} + 2 \sum M'_{AB} + 2 M'_{BA}$$

from Joint equilibrium $M_A = 0$

$$\sum M_{FAB} + 2 \sum M'_{AB} + \sum M'_{BA} = 0$$

$$\sum M'_{AB} = -\frac{1}{2} (\sum M_{FAB} + \sum M'_{BA}) \quad \text{--- (1)}$$

from, $2 M'_{AB} = \frac{4 E I \theta_A}{L} = (K_{AB}) \theta_A$

$$M'_{AB} = \left(\frac{K_{AB}}{2} \right) \theta_A$$

$$\therefore \sum M'_{AB} = \left(\frac{\sum K_{AB}}{2} \right) \theta_A = \frac{1}{2} \theta_A \sum K_{AB}$$

for all the members meeting at A θ_A is same.

$$M'_{AB} = \left(\frac{K_{AB}}{2} \right) \theta_A$$

Putting the ratio, $\frac{M'_{AB}}{\sum M'_{AB}} = \frac{K_{AB}}{\sum K_{AB}}$

$$\therefore M'_{AB} = \frac{(K_{AB})}{(\sum K_{AB})} \sum M'_{AB}. \quad \text{--- (2)}$$

from (1): $M'_{AB} = -\frac{1}{2} \left(\frac{K_{AB}}{\sum K_{AB}} \right) (\sum M_{AB} + \sum M'_{BA})$

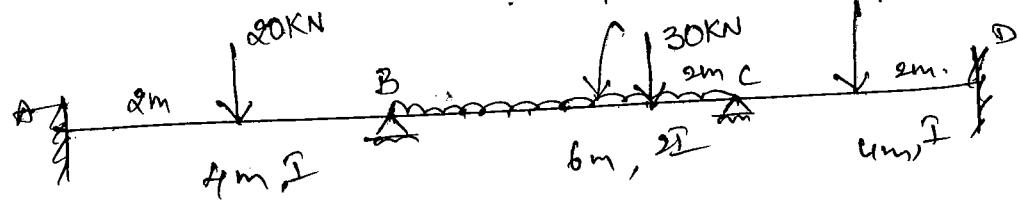
$\therefore -\frac{1}{2} \left(\frac{K_{AB}}{\sum K_{AB}} \right)$ is "Rotation factor"

procedure :-

1. find fixed end moments at all joints
2. calculate near end rotation contribution by assuming far end contribution as 0.
3. Similarly, calculate rotation contribution at all joints which completes first cycle
4. Now rotation contributions are available at far ends. Using the eqn ③ calculate near end contributions at all joints to complete 2nd cycle
5. Repeat the procedure till a change in the rotation contribution of two successive iterations are negligible
6. calculate fixed end moments.

Ques: Analyse the continuous beam using Kanis method.

3A-R
IV-③



FEMs:- $M_{FAB} = -\frac{PL}{8} = -10 \text{ kNm}; M_{FBA} = +10 \text{ kNm}$

$$M_{FBC} = -45 - 13.33 = -58.33 \text{ kNm}$$

$$M_{FCB} = 45 + 26.67 = 71.67 \text{ kNm}$$

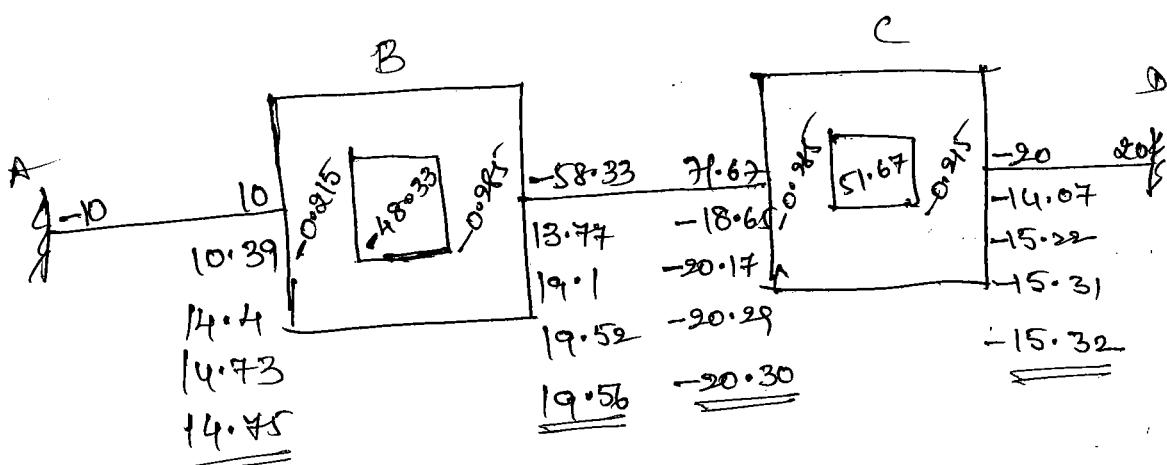
$$M_{FCD} = -80 \text{ kNm}; M_{FDC} = 80 \text{ kNm}$$

$$M_{FCD} = -80 \text{ kNm}; M_{FDC} = 80 \text{ kNm}$$

$$R.F = -\frac{1}{2} \left(\frac{K}{\sum K} \right)$$

Rotation factor:-

Jt	Membr	K	ΣR	R.F
B	BA	$\frac{4EI}{4} = EI$	$2.33EI$	-0.215
	BC	$\frac{4EI(2)}{6} = 1.33EI$		-0.285
	CB	$\frac{4EI(2)}{6} = 1.33EI$		-0.285
C	CD	EI	$2.33EI$	-0.215



$$M_{AB}^1 = R.F \left(\sum M_{FAB} + \sum M_{BA}^1 \right)$$

$$\text{cycle I} \leftarrow M_{BA} = -0.215 (-48.33 + 0) = 10.25$$

$$M_{BC}^I = -0.285 (-48.33 + 0) = 13.77$$

$$M_{CB}^I = -0.285 (51.67 + 13.77) = -18.65$$

$$M_{CD}^I = -0.215 (51.67 + 13.77) = -14.07$$

$$\text{cycle II} \leftarrow M_{BA}^I = -0.215 (-48.33 + 0 - 18.65) = 14.4$$

$$M_{BC}^I = -0.285 (-48.33 + 0 - 18.65) = 19.1$$

$$M_{CB}^I = -0.285 (51.67 + 19.1 + 0) = -20.17$$

$$M_{CD}^I = -0.215 (51.67 + 19.1 + 0) = -15.92$$

$$\text{cycle III} \leftarrow M_{BA}^I = -0.215 (-48.33 + 0 - 20.17) = 14.43$$

$$M_{BC}^I = -0.285 (-48.33 + 0 - 20.17) = 19.52$$

$$M_{CB}^I = -0.285 (51.67 + 19.52 + 0) = -20.29$$

$$M_{CD}^I = -0.215 (51.67 + 19.52 + 0) = -15.31$$

Final End moments $M_{AB} = M_{FAB} + 2M_{BC}^I + M_{BA}^I$

$$M_{AB} = -10 + 2(0) + 14.43 = 4.43 \text{ KNm.}$$

$$M_{BC} = -10 + 2 \times 14.43 + 0 = 39.5 \text{ KNm.}$$

$$M_{BA} = 10 + 2 \times 14.43 + 0 = 39.51 \text{ KNm}$$

$$M_{BC} = -58.33 + 2(19.52) - 20.3 = -39.51 \text{ KNm}$$

$$M_{CB} = 51.67 + 2(-20.3) + 19.52 = 50.63 \text{ KNm}$$

$$M_{CD} = -20 + 2(-15.31) + 0 = -50.64 \text{ KNm}$$

$$M_{DC} = 20 + 2(0) - 15.31 = 4.68 \text{ KNm.}$$

$$M_{DC} = 20 + 2(0) - 15.31 = 4.68 \text{ KNm.}$$

Continuous beam with hinge ends.

SA-II

II-④

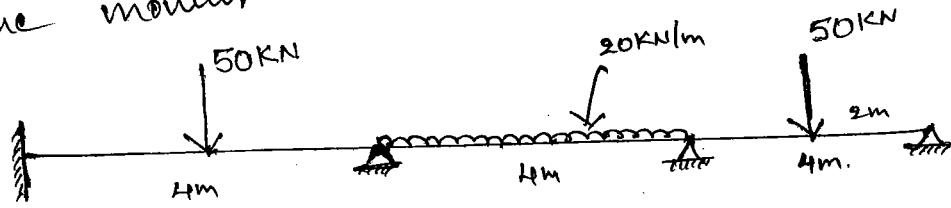
- Stiffener of member with far end Hinge is $\frac{3EI}{4}$.
- far end contribution from hinge is zero.
- Modify the moments of the member AB if B is hinge.

e.g.: - $M_{FAB} = M_{FAB} + \text{com. from } B$

$$M_{FBA} = M_{FBA} - M_{FBA} = 0$$

i.e., Balance the hinge end and carry the moment to the far end

Prob:-



Ans: $M_{FAB} = -25$; $M_{FBA} = 25 \text{ kNm}$; $M_{FBC} = -26.67$; $M_{FCB} = 26.67$

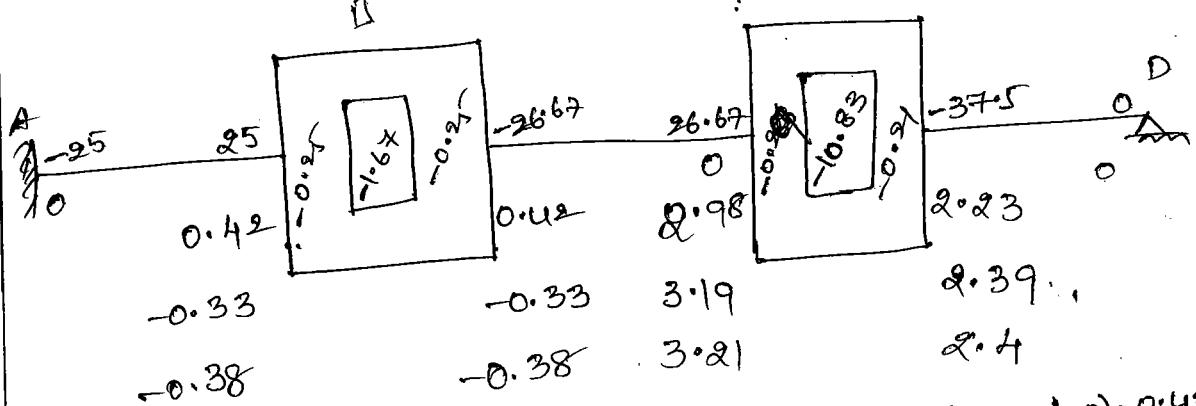
$M_{FCD} = -25$; $M_{FDC} = 25$

modified moments :- $M_{FDC} = 25 - 25 = 0$

$M_{FCD} = -25 + \frac{25}{2} = -37.5 \text{ kNm}$

R.F Table

Jt	Member	K	EI	R.F
A	BA	$\frac{4EI}{4} = EI$		-0.25
B	BC	$\frac{4EI}{4} = EI$	0.25	-0.25
C	CB	$\frac{4EI}{4} = EI$	1.75EI	-0.29
D	CD	$\frac{3EI}{4} = 0.75EI$		-0.21



cycle 2

$$M_{BA}^I = R.F \left(\sum M_{FBA} + \sum M_{AB}^I \right) = -0.25 (-1.67 + 0) = 0.42$$

$$M_{BC}^I = R.F \left(\sum M_{FBC} + \sum M_{BC}^I \right) = -0.25 (-1.67) = 0.42$$

$$M_{CB}^I = R.F \left(\sum M_{FCB} + \sum M_{BC}^I \right) = -0.25 (-10.83 + 0.42 + 0) = 2.98$$

$$M_{CD}^I = -0.21 (-10.83 + 0.42 + 0) = 2.23.$$

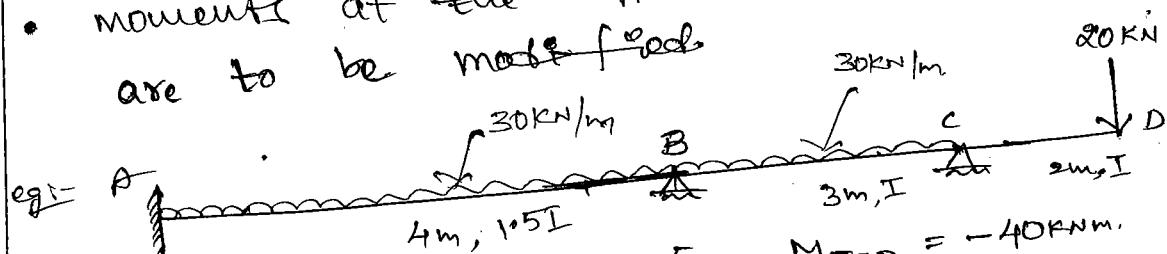
any two iterations gives almost same
non contribution

Why, can we tell any two successive iterations gives almost same solution contributions

	A	B	C	D	E	F	G
Remaining F B.M.	-25	95	-36.67	85.67	-37.5	0	0
Near end (M'_{AB})	0	-0.76	-0.36	6.042	4.8	0	0
far end (M'_{BC})	-0.38	0	3.21	-0.38	0	0	0
Final end moments	-25.38	24.24	-34.24	32.71	-32.71	0	0

Continuous beams with overhangs :- $K_{Bc} = \frac{3EI}{L}$

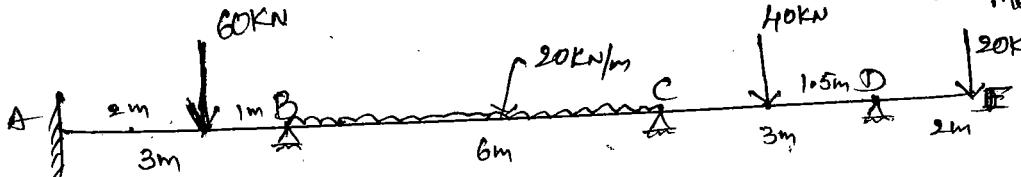
- moments at the support of overhanging portion
are to be ~~more~~ ^{less} good.



$M_{FBC} = -22.5$; $M_{FCB} = 22.5$ $M_{ED} = -40 \text{ kNm}$
 for, $M_{FCB} + M_{CD} = 0 \Rightarrow$ we need to add 17.5 kNm to
 M_{FCB} , so that it will become 40 kNm . Further
 $(17.5/2) = 8.75 \text{ kNm}$ will be carried to M_{FBC} , hence
 M_{FBC} will be modified as -13.75 kNm .

Prob Analyse the continuous beam shown by Kanis method.

SA-II
Q - 5



$$M_{FAB} = -13.33$$

$$M_{FBC} = -60$$

$$M_{FCD} = (-15)$$

$$M_{DE} = -40$$

$$M_{FBA} = 26.67 \text{ kNm}$$

$$M_{FCB} = 60$$

$$M_{FDC} = (-15)$$

$$\text{modified moments}$$

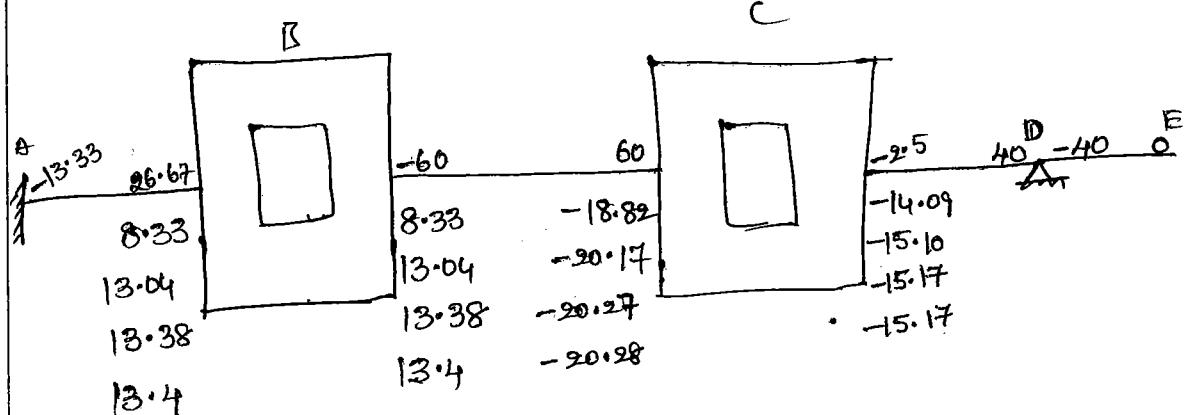
$$M_{FDC} = 15 + 25 = 40$$

$$M_{FCD} = -15 + \left(\frac{25}{2}\right)$$

$$= -2.5 \text{ kNm.}$$

$$R.F BA = -0.25; CB = -0.29$$

$$BC = -0.25; CD = -0.21$$



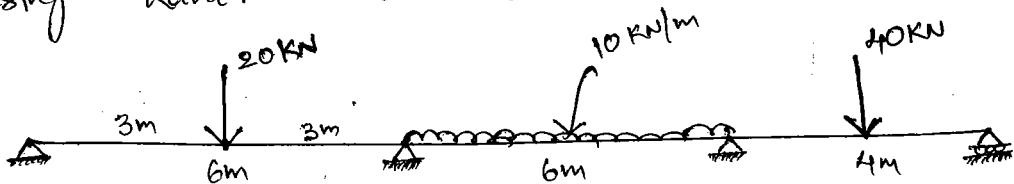
Find end moments:-

Remarks	A	B	C	D	E
FEM1	-13.33	26.67	-60	60	-2.5
$2 \times \text{NEC}$	2×0	2×13.04	2×13.04	2×-20.27	2×-15.17
FEC	13.4	0	-20.28	13.4	0
Final End Moments	0	53.44	-53.44	32.84	-32.84

continuous beam with sinking of supports:-

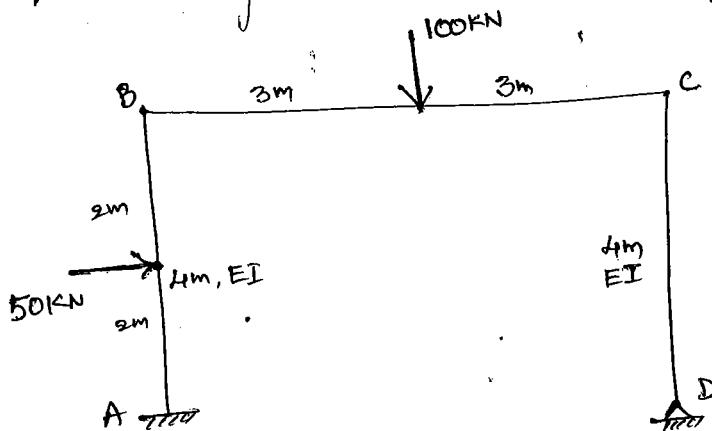
JNTUK prob A Continuous beam ABCD SS over each support consisting of spans AB, BC & CD of length 3m, 4m & 3m resp. Beam carries UDL of 12kN/m over the whole length. Supports B & C sink by 2mm and 7mm resp. Det the moments over each support by using Kanis Method. Take $E = 2 \times 10^5 \text{ N/mm}^2$; $I = 4 \times 10^7 \text{ mm}^4$.

using Kani's method?

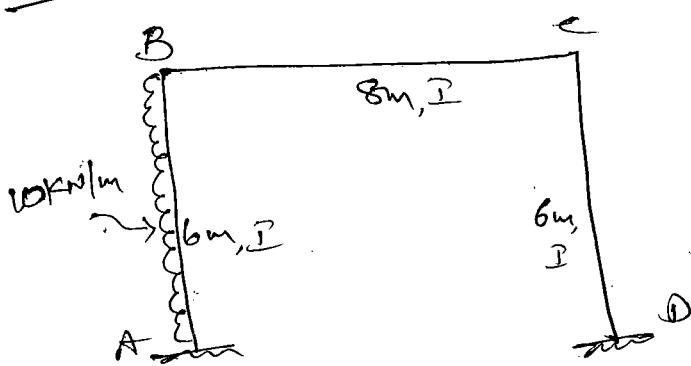


(Solve)
JNTUKT A rectangular portal frame ABCD consists of legs AB and DC of ht 4m and 6m resp. Beam BC of length 8m. carries UDL of 12kN/m and a point load of 80kN at centre. Legs are fixed at their bottom. $I_{ab} : I_{bc} : I_{cd} = 1:2:1$. Analyse the frame using Rotation Contribution Method?

(Solve)
JNTUKT Analyse the portal frame by using RCM?



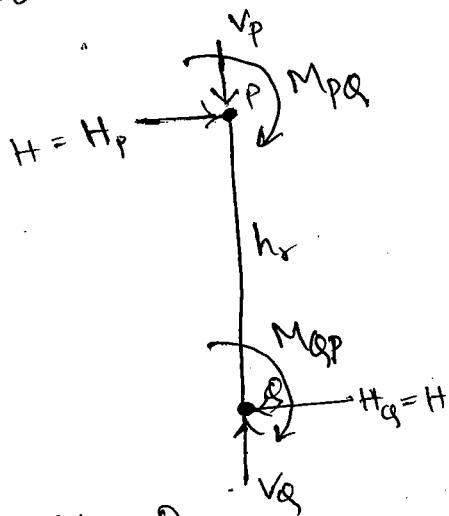
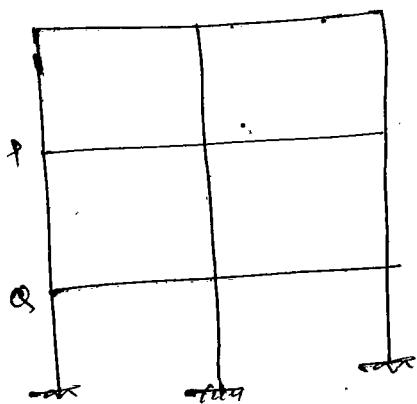
OCT 16:-



frames with sway. (i) varying at same height
 (ii) " " " different.

(i) frame with sway having same column height:

consider a frame shown below, 8th storey.



$$\sum M_P = 0 \quad H \cdot h_r + M_{PQ} + M_{QP} = 0$$

$$H = -\frac{(M_{PQ} + M_{QP})}{h_r}$$

Summing horizontal forces in 8th storey.

$$\sum H = -\frac{(\sum M_{PQ} + \sum M_{QP})}{h_r} = S_r = \text{storey shear}$$

$$\therefore S_r \cdot h_r = -(\sum M_{PQ} + \sum M_{QP}). \quad \text{--- (1)}$$

Let, $\sum M_{PQ}^*$ be the additional moment caused due to sway,

$$\text{final moment, } M_{PQ} + 2 \frac{M_{PQ}}{h_r} + M_{QP} + M_{PQ}^* \quad \text{--- (2)}$$

$$M_{PQ} = M_{FPQ} + 2 \frac{M_{PQ}}{h_r} + M_{QP} + M_{PQ}^* \quad \text{--- (3)}$$

$$M_{QP} = M_{FQP} + 2 \frac{M_{QP}}{h_r} + M_{PQ} + M_{QP}^* = 0$$

If. Loads are zero, $M_{FPQ} + M_{FQP}$ no load condition.

Adding (2) & (3) and taking no load condition,

$$M_{PQ} + M_{QP} = 3 \frac{M_{PQ}}{h_r} + 3 \frac{M_{QP}}{h_r} + M_{PQ}^* + M_{QP}^*$$

$$\sum M_{PQ} + \sum M_{QP} = 3 (\sum M'_{PQ} + \sum M'_{QP}) + 2 \sum M^*_{PQ}$$

$$\therefore M^*_{PQ} = M^*_{QP}$$

moments due to Storey are same.

$$\therefore \sum M^*_{PQ} = \frac{1}{2} (\sum M_{PQ} + \sum M_{QP}) - \frac{3}{2} (\sum M'_{PQ} + \sum M'_{QP}) \quad \text{④}$$

from ① put to ④:

$$\sum M^*_{PQ} = -\frac{S_r h_r}{2} - \frac{3}{2} (\sum M'_{PQ} + \sum M'_{QP})$$

$$\sum M^*_{PQ} = -\frac{3}{2} \left[\frac{S_r h_r}{3} + (\sum M'_{PQ} + \sum M'_{QP}) \right]$$

$$\text{But } M^*_{PQ} = \frac{-6EI\Delta}{L^3} = \frac{-6 \times \frac{1}{3} EI\Delta}{AL \times L} \\ = -1.5 K \left(\frac{\Delta}{L} \right) \quad \text{⑤}$$

$$\sum M^*_{PQ} = -1.5 \left(\frac{\Delta}{L} \right) \Sigma K$$

$$\frac{M^*_{PQ}}{\sum M^*_{PQ}} = \frac{K}{\Sigma K} \Rightarrow M^*_{PQ} = \left(\frac{K}{\Sigma K} \right) \sum M^*_{PQ}$$

$$\therefore M^*_{PQ} = \frac{-3}{2} \left(\frac{K}{\Sigma K} \right) \left[\frac{S_r h_r}{3} + (\sum M'_{PQ} + \sum M'_{QP}) \right]$$

" " is called Displacement factor.

$\frac{-3}{2} \left(\frac{K}{\Sigma K} \right)$ is called Storey moment.

$\frac{S_r h_r}{3}$ is called

" " Storey shear can be found by taking sum of all horizontal forces above the section through the storey in consideration

$$M_{PQ}^* = D \cdot F \left[\frac{S_r h_r}{3} + (M_{PQ}^I + M_{QP}^I) \right]$$

SA-II
V-7

Dip. Contribution = $D \cdot F \left[\text{Storey moment} + \sum (\text{Rotation contribution at top and bottom ends of column in that storey}) \right]$

$$M_{AB}^I = D \cdot F (M_{FAB}^I + M_{BA}^I + M_{PQ}^I)$$

columns with different height :- (hr)
choose a convenient height as storey height.

$$\sum M_{PQ}^I \cdot \left(\frac{h_r}{h_{PQ}} \right) = -\frac{3}{2} \left[\frac{S_r h_r}{3} + (M_{PQ}^I + M_{QP}^I) \right]$$

Denoting : $\frac{h_r}{h_{PQ}} = C_{PQ}$

$$\sum C_{PQ} M_{PQ}^I = -\frac{3}{2} \left[\frac{S_r h_r}{3} + \sum C_{PQ} (M_{PQ}^I + M_{QP}^I) \right]$$

$$M_{PQ}^I = \frac{-G E I \Delta}{h_{PQ}^2} = \frac{-1.5 K \Delta}{h_{PQ}} \Rightarrow M_{PQ}^I \propto \frac{K}{h_{PQ}}$$

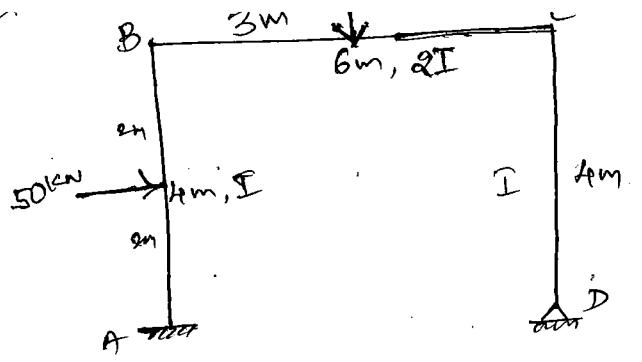
$$M_{PQ}^I \propto K \left(\frac{h_r}{h_{PQ}} \right) \propto \cancel{h_{PQ}} \quad (\because h_r \text{ is const})$$

$$\frac{M_{PQ}^I}{\sum M_{PQ}^I} = \frac{K C_{PQ}}{\sum C_{PQ}^2 K}$$

$$\therefore M_{PQ}^I = \frac{3}{2} \left[\frac{K C_{PQ}}{\sum C_{PQ}^2 K} \left(\frac{S_r h_r}{3} + (M_{PQ}^I + M_{QP}^I) C_{PQ} \right) \right]$$

$$D \cdot F = \frac{3}{2} \left(\frac{K C_{PQ}}{\sum K C_{PQ}^2} \right)$$

$$D \cdot C = D \cdot F \left[\frac{S_r h_r}{3} + \sum (M_{PQ}^I + M_{QP}^I) C_{PQ} \right].$$



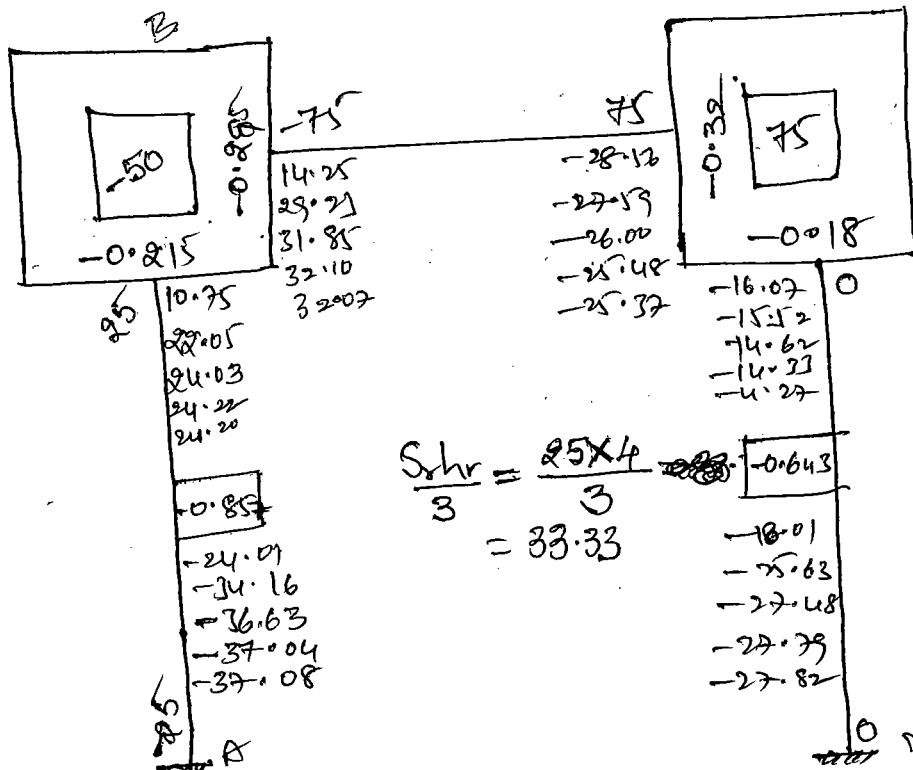
$$\begin{aligned} M_{FAB} &= -25 \text{ KNm} \\ M_{FBA} &= 25 \text{ KNm} \\ M_{FBC} &= -25 \text{ KNm} \\ M_{FCB} &= 25 \text{ KNm} \\ M_{FCD} &= M_{PDC} = 0 \end{aligned}$$

R.F. Taddei

R.O.F		members	K	
B	BA	$\frac{4EI^2}{4} = EI^2$		-0.215
	BC	$\frac{4Ex^2I}{6} = 1.33 EI^2$	$\alpha = 33 EI^2$	-0.885
C	CB	1.33 EI^2		-0.32
	CD	$\frac{3EI^2}{4} = 0.75EI^2$	$\alpha = 0.083 EI^2$	-0.18

Disp. factor Table

<u>Memb</u>	<u>K</u>	<u>ΣK</u>	-0.857
AB	EI	1.35EI	-0.643
CD	0.75EI		



$$\text{Rot. Contr. } M_{AB}^I = R.F \left(\sum M_{FAG} + \sum M_{OB}^I + \sum M_{PQ}^* \right)$$

$$\text{Disp. Contr. } M_{PQ}^* = D.F \left(\frac{S_r \cdot h_r}{3} + (\sum M_{PQ}^I + \sum M_{QP}^I) \right)$$

Cycle I :- $M_{BA}^I = -0.215(-50+0+0+0) = 10.75$

$$M_{BC}^I = -0.215(-50+0+0+0) = 10.75$$

$$M_{CB}^I = -0.32(75+10.75+0+0) = -28.56$$

$$M_{CD}^I = -0.18 (\quad \quad \quad) = -16.07$$

$$M_{AB}^* = -0.857(33.33 + 10.75 + 0 - 16.07 + 0) \\ = -24.01$$

$$M_{CD}^* = -0.643 (\quad \quad \quad) = -18.01$$

likewise for all cycles.

Final End moments:-

$$M_{AB} = -25 + 10 + 10 - 37.08 = -37.88$$

$$M_{BA} = 25 + 10 - 37.08 = 36.32$$

$$M_{BC} = -75 + 10 - 25.37 = -36.23$$

$$M_{CE} = -75 + 10 - 25.37 = -25.37$$

My

If columns have different length

D.F Table

Storey	Member	K	C _{pl}	$\sum K C_{pl}^2$	D.F
1	AB	$\frac{4E^2}{L}$	$\frac{3}{3} = 1.0$	1.08755	-1.055
	CD	$\frac{4E^2}{L}$	$\frac{3}{4} = 0.75$		-0.5

$$C_{pl} = \frac{h_r}{h_{pl}} ; D.F = \frac{K C_{pl}}{\sum K C_{pl}}$$

SA-II

IV-⑧

Different Column Cases

prob A rectangular portal frame ABCD consists of legs AB and DC of ht 4m and 6m resp. Beam BC of length 8m. Beam carries UDL of 14 KN/m and a point load at centre of 80KN. Legs are fixed at their bottom. I_{ab}: I_{bc}: I_{cd} = 1:2:1. Analyse the frame using rotation curve method.

$$\text{Ans} \quad M_{FAB} = 0 = M_{FBA}$$

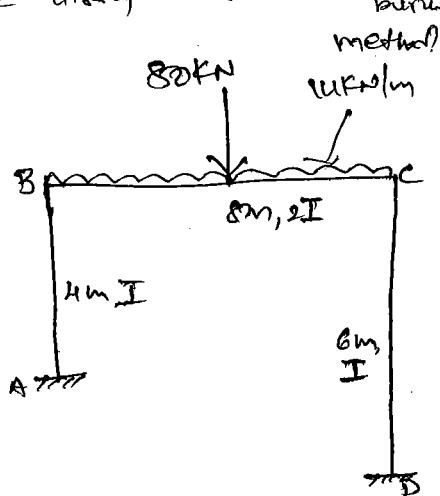
$$M_{FBC} = -74.67 - 80 = -154.67$$

$$M_{FCB} = 154.67$$

$$M_{FCD} = 0 = M_{FDC}$$

R.F Taken

<u>Jt</u>	<u>Memb</u>	<u>K</u>	<u>ΣK</u>	<u>R.F</u>
A	BA	$\frac{4EI}{4} = EI$	<u>ΣK</u>	
B	BC	$\frac{4Ea^2I}{8} = EA$	<u>ΣF</u>	

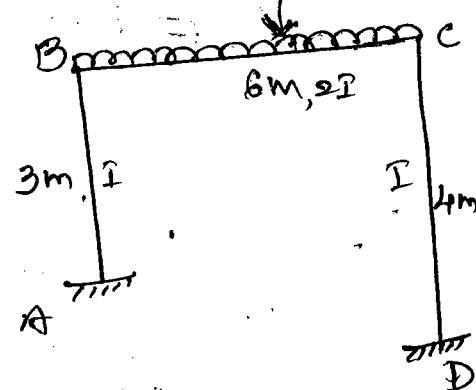


Analysis of frames with Sway when columns in a storey have different heights.

Ex. Analyse the portal frame shown below by Kanit's method?

$$\text{Ans. } \text{FEM}_A = M_{FAB} = M_{FBA}$$

$$= M_{FCD} = M_{FDC} = 0$$



$$M_F BC = -120 \text{ kNm}$$

$$M_F CD = +120 \text{ kNm}$$

R.F	J.F	Memb	K	ΣK	R.F
	B	BA	$\frac{4EI}{3}$		-0.25
	C	BC	$\frac{4EI(2)}{6} = \frac{4EI}{3}$	$\frac{8EI}{3}$	-0.25
		CB	$\frac{4EI(2)}{6} = \frac{4EI}{3}$		-0.286
		CD	$\frac{4EI}{4} = EI$	$\frac{7EI}{3}$	-0.214

$$\text{D.F} = -\frac{3}{2} \left(\frac{K_{CPB}}{\sum K_{CP}} \right) \quad \text{Let } h_B = 3 \text{ m}$$

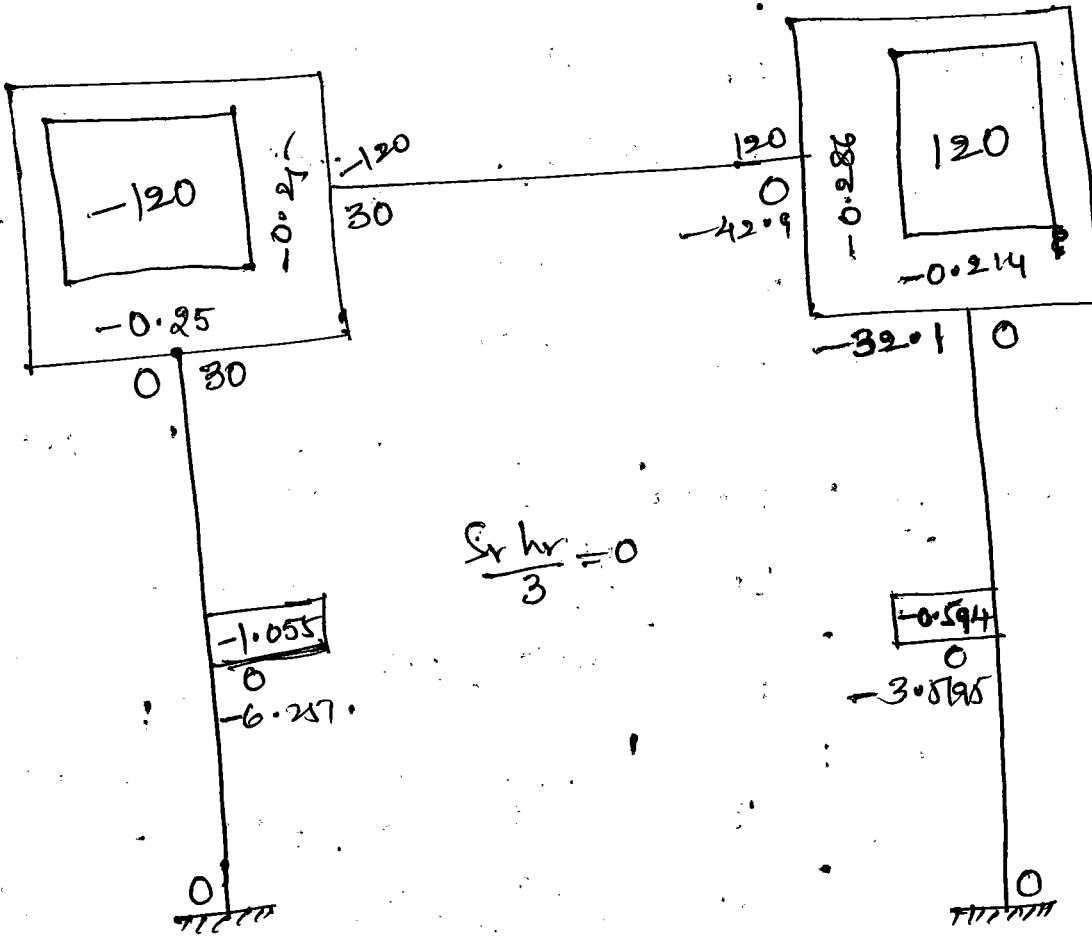
(ref. h_B)

$$C_{AB} = \frac{h_B}{h_B + h_{CD}} = \frac{3}{3} = 1.0 ; C_{CD} = \frac{3}{4} = 0.75$$

Storey	Memb	K	C_{PB}	K_{CPB}	$\frac{1}{2} K_{CPB}$	D.F
1	AB	$\frac{4EI}{3} = 1.33EI$	1.0	$1.33EI$	$0.665EI$	-1.055
	DC	$\frac{4EI}{4} = EI$	0.75	$0.75EI$	$0.375EI$	-0.594

Total D.F. need to be taken
Displacement Contribution must be
modified as,

$$M_{pq}^* = D.F. \left[\frac{S_r h_r}{3} + \sum (M_{p\bar{s}}^l + M_{q\bar{s}}^l) C_{pq} \right]$$



$$\frac{S_r h_r}{3} = 0$$

C-II $M_{BA}^l = -0.25 (-120 + 0 + 0 + 0) = 30$

$M_{BC}^l = " = 30$

$M_{CB}^l = -0.286 (120 + 30 + 0 + 0) = -42.9$

$M_{CD}^l = -0.214 (" ") = -32.1$

$M_{AB}^* = -1.055 (0 + (0+30) + (0-32.1) 0.75 = -6.25)$

$M_{CD}^* = -0.594 (0 + " ") = -3.5195$

C-II $M_{BA}^l = -0.25 (-120 + 0 - 42.9 - 6.25) =$

-- Continue --

UNIT - VI

SA-II

VI-(I)

Introduction to Matrix Methods

Flexibility Methods: Introduction, Application to Continuous beams
Stiffness " " " " "

- Analysis of Indeterminate Structures is the major field in Structural Engg.
- Kani's Method is the best one among several methods of Frame Analysis. But this also will not be convenient for present day multi-storey buildings.
- Hence there is a need for Matrix Method.

Flexibility Matrix Method:- (forces) Compatibility method

- Systematic development of consistent deformation method in matrix form has lead to the flexibility matrix method
- The unknowns are forces, In this method
- Identifying basic determinate structure and thereby identifying redundant forces is the key feature
- No. of Redundant forces is equal to the Degree of Static Indeterminacy
- Displacements in basic determinate str. due to given loads and Redundants are found and conditions of consistency are formed

Approaches to Matrix Methods

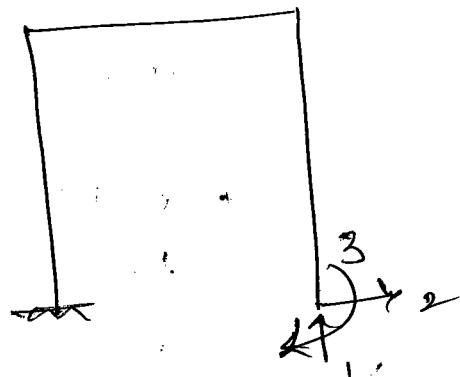
(i) Direct / Structure

(ii) Transformation matrix / Element.

Generalized Coordinate System-

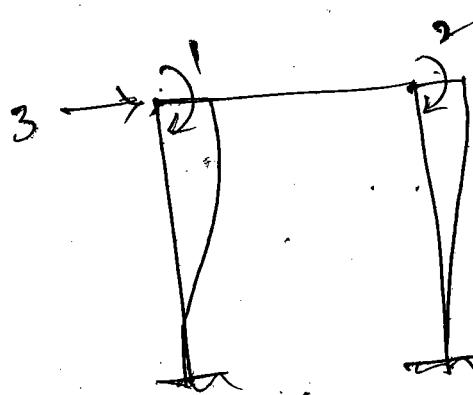
The directions of forces and disp. to determine the unknowns in the str. Systems are known as generalized coordinates.

e.g.



$$\{F\} = \begin{Bmatrix} V_D \\ H_D \\ M_D \end{Bmatrix}$$

$$\{\Delta\} = \begin{Bmatrix} \Delta V_D \\ \Delta H_D \\ \Delta M_D \end{Bmatrix}$$



$$\{F\} = \begin{Bmatrix} M_B \\ M_C \\ S \end{Bmatrix}$$

$$\{\Delta\} = \begin{Bmatrix} O_B \\ O_C \\ S \end{Bmatrix}$$

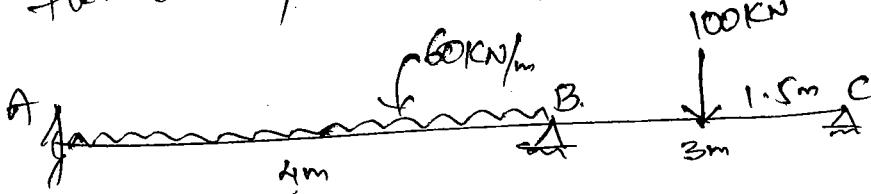
flexibility Matrix of a Structure

has 'n' no. of coordinates, its replacement response to the forces is represented by, $\{S\} = \begin{bmatrix} f_1 & f_2 & \dots & f_n \\ f_{11} & f_{12} & \dots & f_{1n} \\ f_{21} & f_{22} & \dots & f_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{n1} & f_{n2} & \dots & f_{nn} \end{bmatrix}$

which is known as flexibility matrix $[f_{ij}]$

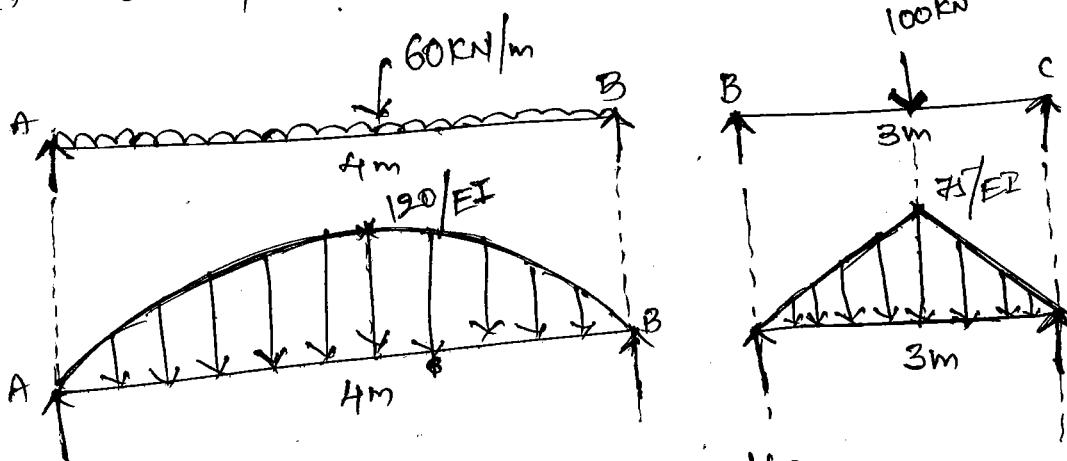
- Element S_{ij} represent, Displ. at coordinate $sA-II$
 due to unit force at j . VI - ②
- To develop flexibility matrix, a unit force is applied successively at coordinates $1, 2, 3, \dots, n$ and displacements at all coordinates are computed
- From Maxwell's Reciprocal Theorem
 $S_{ij} = S_{ji}$, Hence flexibility matrix has diagonal symmetry.
- flexibility matrix method :- procedure
- (i) determine degree of static indeterminacy
 - (ii) choose the redundants and assign coordinates to them
 - (iii) remove the redundants and prepare basic determinate structure
 - (iv) determine deflections in the directions of coordinates due to given loads
 - (v) determine the elements in flexibility matrix by applying unit force at each coordinate and finding disp. at all the coordinates.
 - (vi) apply the compatibility condition to compute $\{P\} = [f]^{-1} \{ \Delta \} - \{ \Delta_L \} \}$
 - (vii) knowing the redundant forces compute the member forces like SF Ee BM,

prob:- Analyse the continuous beam by flexibility method?



~~step ①~~ D.S.I. $\rightarrow \alpha$

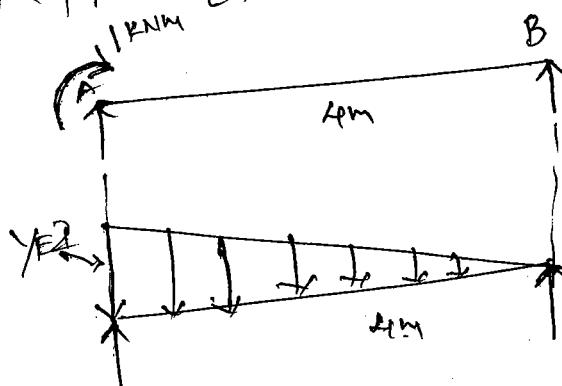
~~step ②~~ choosing the redundance as $M_{AC} M_B$



$$\Delta_{IL} = \phi_A = SF_A = \left(\frac{q}{g} \times 4 \times \frac{120}{EI} \right) \times \frac{1}{2} = \frac{160}{EI}$$

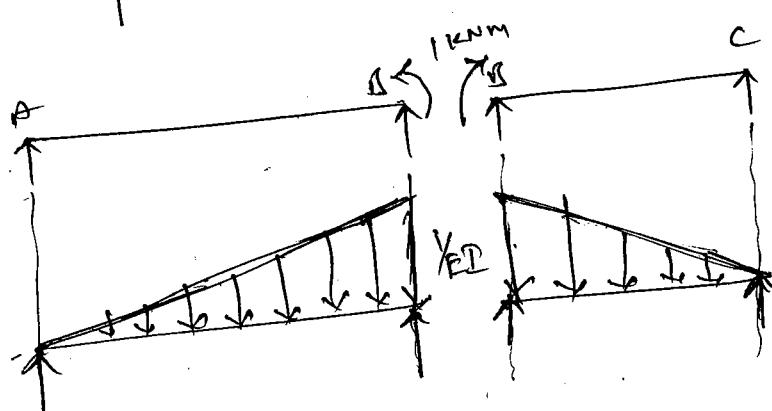
$$\Delta_{\alpha L} = \frac{160}{EI} + \left(\frac{1}{2} \times 3 \times \frac{75}{EI} \right) \times \frac{1}{2} = \frac{216.25}{EI}$$

~~to find [f] matrix.~~



$$f_{11} = \phi_A = \left(\frac{1}{2} \times 4 \times \frac{1}{EI} \right) \times \frac{\frac{q \times 4}{4}}{4} \\ = \frac{q \times q}{3EI} = \frac{4}{3EI}$$

$$f_{21} = \frac{q}{3EI}$$



$$f_{12} = \left(\frac{1}{2} \times 4 \times \frac{1}{EI} \right) \times \frac{q}{3} \\ + \left(\frac{1}{2} \times 3 \times \frac{1}{EI} \right) \times \frac{2}{3} \\ = \frac{7}{3EI}$$

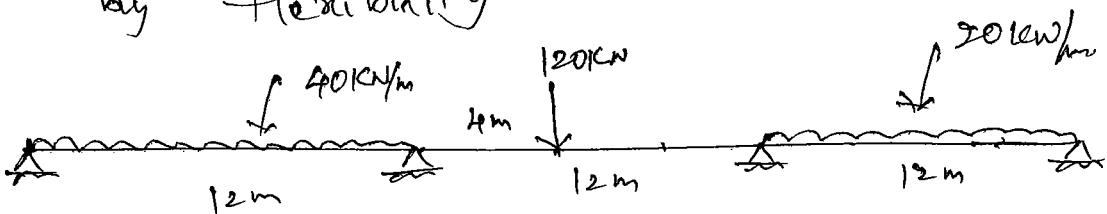
$$B.C's \quad \{\Delta\} = \{0\}; \quad \{\Delta_L\} = \left\{ \begin{array}{l} \frac{160}{EI} \\ \frac{216.25}{EI} \end{array} \right\}$$

$$[f] = \begin{bmatrix} \frac{4}{3EI} & \frac{2}{3EI} \\ \frac{2}{3EI} & \frac{7}{3EI} \end{bmatrix} = \frac{1}{3EI} \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}$$

$$\therefore \{P\} = [f]^T \{(\Delta_3 - \{\Delta_L\})\}$$

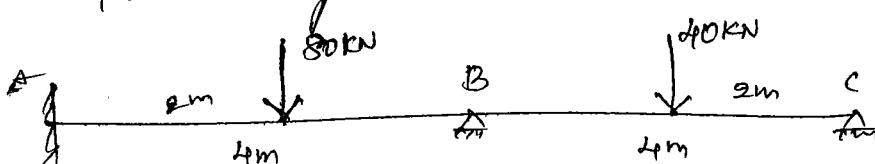
$$\therefore \{P\} = \begin{Bmatrix} M_A \\ M_B \end{Bmatrix} = \begin{Bmatrix} -85.875 \\ -68.25 \end{Bmatrix} \text{ KNm}$$

prob:- Analyse the continuous beam shown below by flexibility matrix method



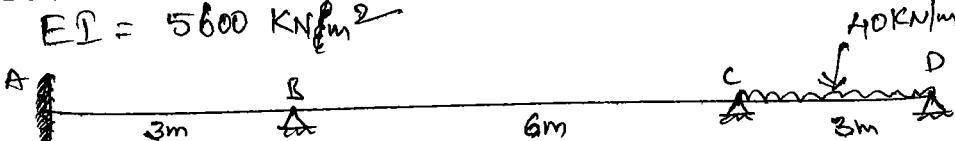
soln $M_B = -4449.78 \text{ KNm}; \quad M_C = -174.92 \text{ KNm}$

prob:- Analyse the continuous beam shown, if the downward settlement of supports B and C are 10mm and 5mm resp. Take $EI = 184 \times 10^6 \text{ Nmm}^2$. Use flexibility method?



soln $\{\Delta\} = \begin{Bmatrix} -0.01 \\ -0.005 \end{Bmatrix}; \quad \{P\} = \begin{Bmatrix} R_B \\ R_C \end{Bmatrix} = \begin{Bmatrix} 18.485 \\ 27.105 \end{Bmatrix}$

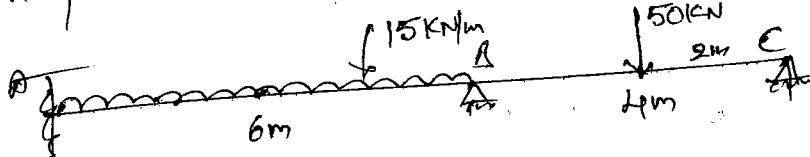
prob(4):- Solve the beam shown below, if $\Delta_B = 30\text{mm} (\downarrow)$
 $EI = 5600 \text{ KNm}^2$



soln $M_A = -84.81 \text{ KNm}, \quad R_B = -62.24 \text{ KN}, \quad M_C = -43.14 \text{ KNm}$

Prob:- Analyse the continuous beam by force method.

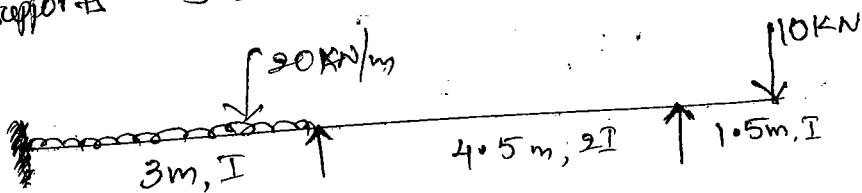
(5)



$$M_A = -65.29 \text{ kNm} \quad ; \quad M_B = -49.41 \text{ kNm}$$

Sol:-

prob(6):- Analyse the continuous beam Using Compatibility method if the downward Settlements of supports B & C are $100/EI$ and $50/EI$ resp.



Stiffness Matrix Method (Dis. methods Eqn method)

SA-II

VI-4

- In this method, basic unknowns are displacements.
- Hence, the degree of kinematic Indeterminacy is identified.
- To start with, Joint displacement in all directions are restrained
- Systematic development of slope deflection method in the matrix form has given rise to Stiffness matrix method
- Equations of equilibrium are formed and solved for slopes and deflections at the joints.

Stiffness matrix If a structure is having n coordinates its force response to the displacement is represented by,

$$\begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & K_{2n} \\ \vdots & & & \\ K_{n1} & K_{n2} & \dots & K_{nn} \end{bmatrix}$$

which is called as Stiffness matrix.

- K_{ij} is the force at 'i' due to a unit disp. at 'j'
- To develop stiffness matrix, unit displacement should be given successively at coordinates 1, 2, ..., n and the force developed at all the coordinates are computed
- from Maxwell's reciprocal theorem $K_{ij} = K_{ji}$
Hence, Stiffness matrix has diagonal symmetry.

Relation b/w flexibility and stiffness

$$\{\Delta\} = [f] \{P\} \Rightarrow \text{from } f = \frac{\Delta}{P}$$

$$\{P\} = [K] \{\Delta\} \Rightarrow K = \frac{P}{\Delta}$$

$$\{P\} = [K] [f] \{P\}$$

$$\therefore [K] [f] = [I] \leftarrow \text{Identity matrix.}$$

Hence flexibility and stiffness matrices are inverse of each other.

- procedure :-
- (i) Det. DOF & Kinematic Indeterminacy
 - (ii) Assign the co-ordinates to unknown disp.
 - (iii) Impose restraints in all coordinate directions to get fully restrained str.
 - (iv) Det. forces developed in each of the coordinate directions of the fully restrained structure called $\{PL\}$.

(v) Develop stiffness matrix $[K]$ by giving unit displ. to the restrained structure in each of the coordinate directions

(vi) write the B.C's $\{P\}$, i.e., total force

in the 1-directions of coordinates

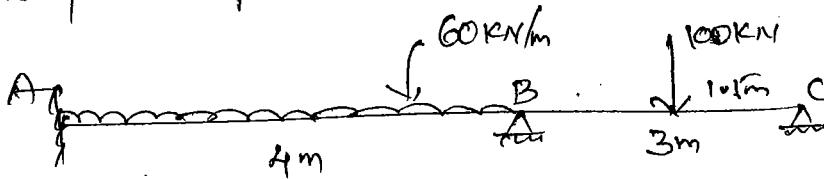
(vii) solve for $\{\Delta\}$, from $\{\Delta\} = [K]^{-1} \{P\} - \{PL\}$

(viii) calculate the member forces
using the joint displacements calculated

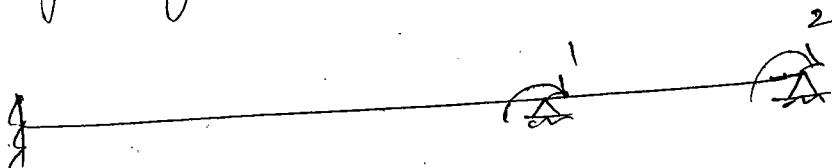
Prob(1):- Analyse the continuous beam shown below using L-spl. method. Take EI is const.

SA-II

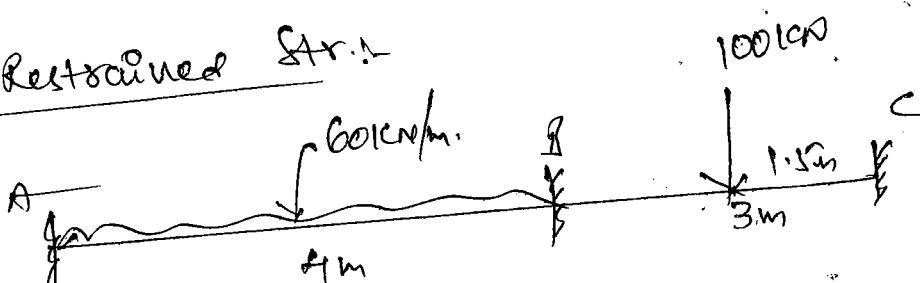
VII - (5)



Ans: neglecting axial deformations $D_K = 2$



fully Restrained Strt.

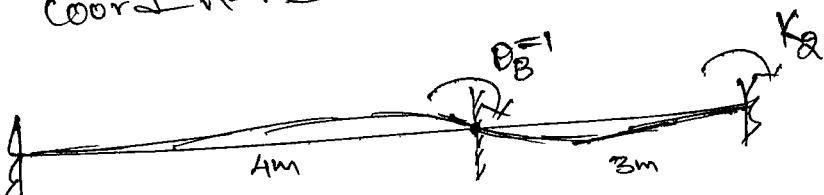


$$M_{FAB} = -80 \text{ kNm}; M_{FBA} = 80 \text{ kNm}$$

$$M_{FBC} = -37.5 \text{ kNm}; M_{FCB} = 37.5 \text{ kNm}$$

$$\{P_E\} = \begin{Bmatrix} P_{E1} \\ P_{E2} \end{Bmatrix} = \begin{Bmatrix} M_B \\ M_C \end{Bmatrix} = \begin{Bmatrix} 80 - 37.5 \\ 37.5 \end{Bmatrix} = \begin{Bmatrix} 42.5 \\ 37.5 \end{Bmatrix}$$

To develop stiffness matrix:- give unit rotation
in coordinate direction i.



$$K_{11} = \frac{4EI}{A} + \frac{4EI}{3} = \frac{7EI}{3}$$

$$K_{21} = \text{force at } \dot{x} \text{ due to L-spl at } i = \frac{2EI}{3}$$



$$K_{22} = \frac{4EI}{3}$$

Final forces acting in coordinate directions
at 1 & 2 are zero. $\{P\} = \{0\}$.

$$[K] = \begin{bmatrix} \frac{7EI}{3} & \frac{2EI}{3} \\ \frac{2EI}{3} & \frac{4EI}{3} \end{bmatrix}$$

$$\{\Delta\} = [K]^{-1} \{ \{P\} - \{P_L\} \}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{3}{EI(7xL - 2^2)} \begin{bmatrix} 4 & -2 \\ -2 & 7 \end{bmatrix} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix} \right\}$$

$$= \begin{Bmatrix} -\frac{11.875}{EI} \\ -\frac{22.188}{EI} \end{Bmatrix}$$

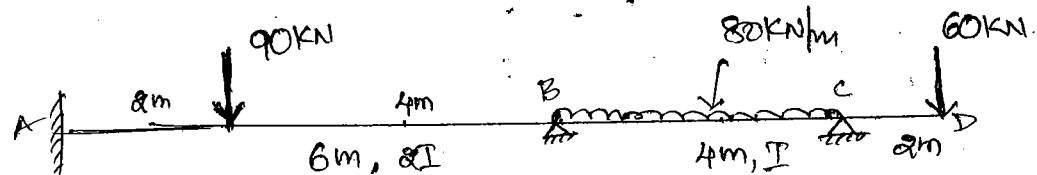
$$M_{AB} = M_{FAB} + \frac{2EI}{L} (2\theta_A + \theta_B) \\ = -80 + \frac{2EI}{A} \left(2 \times 0 - \frac{11.875}{EI} \right) = -85.938 \text{ KNM}$$

$$M_{BA} = 68.13 \text{ KNM}$$

$$M_{BC} = -68.13 \text{ KNM}$$

$$M_{CB} = 0$$

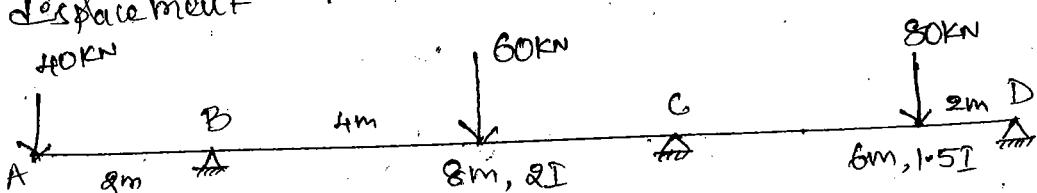
Analyse the beam shown below using stiffness matrix method?



Sol: $\{P\} = \begin{Bmatrix} 0 \\ 120 \end{Bmatrix}; \begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \begin{Bmatrix} \frac{48 \cdot 80}{EI} \\ -1.017 \end{Bmatrix}$

$M_{AB} = -60.8; M_{BA} = 78.403 \text{ kNm}; M_{BC} = -78.403; M_{CB} = 120 \text{ kNm}$

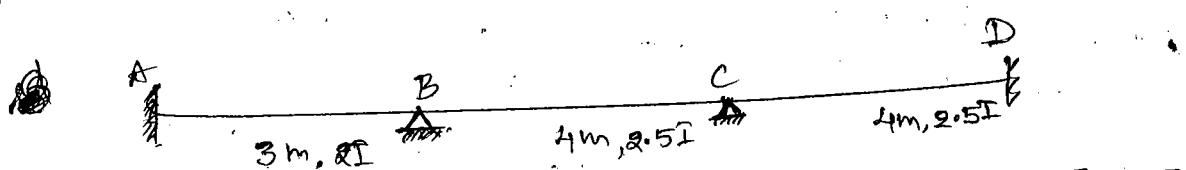
Prob ③:- Analyse the continuous beam shown below by displacement method?



Sol: $\{P\} = \begin{Bmatrix} -80 \\ 0 \\ 0 \end{Bmatrix}; \begin{Bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{Bmatrix} = \begin{Bmatrix} -\frac{27.035}{EI} \\ \frac{14.07}{EI} \\ -\frac{78.145}{EI} \end{Bmatrix}$

$M_{BG} = -80; M_{CB} = 60.55; M_{CD} = -60.55; M_{DC} = 0$

Prob ④:- Analyse the continuous beam shown below if the Support B Sinks by 10mm. Use disp. method. Take $EI = 6000 \text{ KNm}^2$?



Sol: $\begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \begin{Bmatrix} \frac{7.079}{EI} \\ -13.198 \end{Bmatrix}$

$M_{AB} = -69.613; M_{BA} = -59.227$

$M_{BC} = 59.227; M_{CB} = 33 \text{ kNm}$

prob:- Difference between flexibility & stiffness method?

Flexibility Method

- (i) Also called as force method
- (ii) Compatibility method
- (iii) Unknowns are forces
- (iv) Degree of static Indeterminacy is to be calculated
- (v) It is developed on consistent deformation method
- (vi) Consistency conditions are formed and solved to get unknown forces
- (vii) Basic structure is the method of determinate str.
- (viii) D.S.I is the size of flexibility matrix
- (ix) flexibility matrix is developed by applying unit force at each coordinate.
- (x) flexibility matrix has diagonal symmetry.
- (xi) uses generalized coordinate system to solve indeterminate structures
- (xii) f_{ij} represent disp. at i due to unit force at j .
- (xiii) compatibility conditions $\{F\} = [f]^{-1} \{ \{A\} - \{P\} \}$.

Stiffness Method

- (i) Also called as displ. method Equilibrium method.
- (ii) Unknowns are displacements.
- (iii) Degree of kinematic Indeterminacy is to be calculated
- (iv) It is developed on slope-deflection method.
- (v) Equilibrium equations are formed and solved to get unknown disp.
- (vi) Basic str. is fully restrained structure which is indeterminate
- (vii) D.K.I is the size of stiffness matrix.
- (viii) Stiffness matrix is developed by applying unit displacement at each coordinate
- (ix) Stiffness matrix is the inverse of flexibility matrix
- (x) Stiffness matrix has diagonal symmetry.
- (xi) uses generalized coordinate system to analyse indeterminate str.
- (xii) K_{ij} represent force at i due to unit disp. at j . Equilibrium conditions.
- (xiii) $\{A\} = [K]^{-1} \{ \{F\} - \{P\} \}$