

Unit - 4 (part - A)

Laminar flow : —

The Flow of a fluid when each particle of the fluid follows a smooth path, paths which never interfere with one another.

Turbulent flow : —

It is an irregular flow that is characterized by tiny whirlpool regions.

Characteristics Of Turbulent Flow : —

- Turbulent flow tends to occur at higher velocities, low viscosity and at higher characteristic linear dimensions.
- If the Reynolds number is greater than $Re > 3500$, the flow is turbulent.
- The flow is characterized by irregular movement of fluid.
- The process of turbulence is highly unsteady, so that the flow velocity at a given point is shown to great variance over time.
- Turbulence is a three dimensional phenomenon

Characteristics of Laminar flow : —

- It is characterized by smooth streamlines and highly ordered motion.

If the Reynolds number is less than $Re > 2300$, the flow is laminar

Laminar flow is common only in cases in which the flow channel is relatively small, the fluid is moving slowly and its viscosity is relatively high.

- There is no motion in radial direction and the velocity component in the direction normal to flow is zero.
- There is no acceleration since the flow is steady.
- The steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular pipe.

Laws Of Friction :—

There are mainly 5 laws

Whean on ab

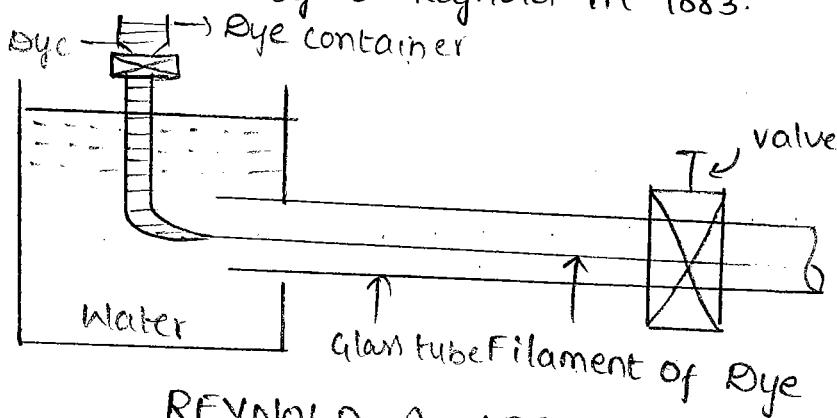
Laminar Flow	Turbulent flow
<ul style="list-style-type: none"> * It is proportional to velocity of flow * Independent of pressure * It is proportional to Area of surface in contact * Independent of nature of surface in contact * Greatly affected by Temperature of variations on flowing fluid 	<ul style="list-style-type: none"> * It is proportional to $(\text{velocity})^n$ n varies from 1.72 to 2.0 * Independent of pressure * It is proportional to density of flow in fluid & area of surface in contact * Independent of nature of surface in contact * Slightly affected by Temperature of variations on flowing liquid.

Reynolds Experiment :-

The type of flow is determined by Reynolds number i.e.

$$Re = \frac{\rho V d}{\mu}$$

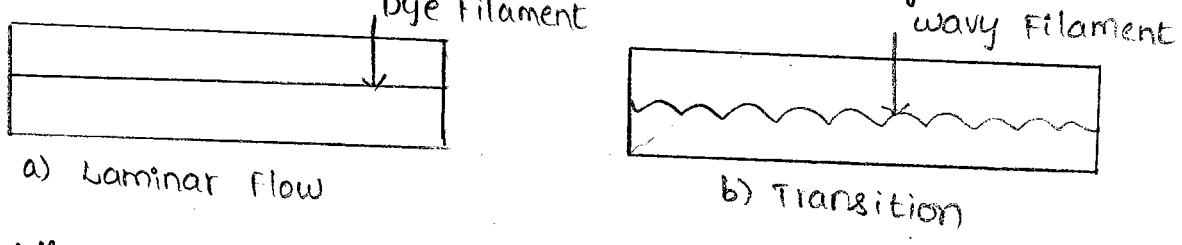
this was demonstrated by O. Reynolds in 1883.



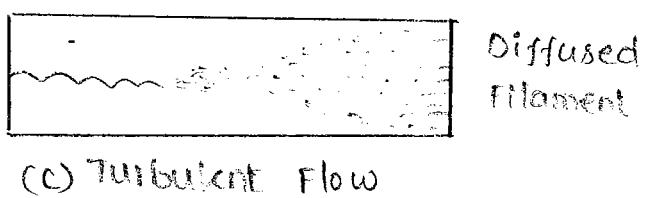
REYNOLD APPARATUS

the following observations were made by Reynold:

- i) When the velocity of flow was low, the dye filament in the glass tube was in the form of a straight line. This straight line of dye filament was parallel to glass tube, which was the case of laminar flow as shown in below fig (a)



- ii) With the increase of velocity of flow, the dye filament was no longer a straight line but it became a wavy nature as shown in above fig (b). This show no longer laminar.
 - iii) With further increase of velocity of flow, the wavy dye-filament broke-up and finally diffused in water as shown in fig (c).



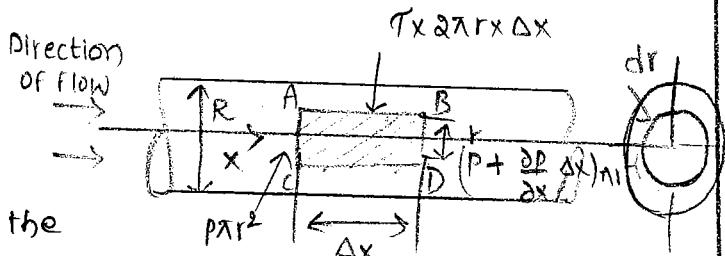
In case of Turbulent flow the mixing of dye-filament and water is intense and flow is irregular, Random & disorderly.

Laminar flow, Loss of head is proportional to the velocity pressure but in case of Turbulent flow, loss of Head is approximately proportional to the square of velocity.

$$\therefore \text{Loss of Head } h_f \propto V^n \quad n \rightarrow 1.95 \text{ to } 2.0$$

SHEAR DISTRIBUTION IN LAMINAR FLOW :

Consider a Horizontal pipe of Radius 'R'. The viscous fluid is flowing from left to right in the pipe as shown in fig. Consider a fluid element of radius 'r' sliding in a cylindrical element of radius $(r+dr)$. Let the length of the fluid element be Δx . If 'P' is the intensity on AB & $(P + \frac{\partial P}{\partial x} \Delta x)$ on CD.



1. The pressure force, $P\pi r^2$ on face AB

2. The pressure force, $(P + \frac{\partial P}{\partial x} \Delta x) \pi r^2$ on face CD

3. Shear force, $\tau \times 2\pi r \Delta x$ on surface of fluid element.

As there is no acceleration summation of all forces must be zero

$$P\pi r^2 - (P + \frac{\partial P}{\partial x} \Delta x) \pi r^2 - \tau \times 2\pi r \Delta x = 0$$

$$-\frac{\partial P}{\partial x} \cdot \Delta x \pi r^2 - \tau \times 2\pi r \Delta x = 0$$

$$-\frac{\partial P}{\partial x} \cdot r - 2\tau = 0$$

$$\therefore \tau = -\frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

(3)

Velocity Distribution in Laminar flow:-

We know that $\tau = \mu \frac{du}{dy}$

y is measured from the pipe wall.

let $y = R - r$ i.e. $dy = -dr$

$$\tau = \mu \frac{du}{-dr}$$

$$\tau = -\mu \frac{du}{dr}$$

As we know from shear stress, we derive $\tau = -\frac{\partial P}{\partial x} \cdot \frac{r}{2}$

$$+\mu \frac{du}{dr} = -\frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{2\mu} \frac{\partial P}{\partial x} \cdot r$$

By Integrating $\Rightarrow u = \frac{1}{4\mu} \frac{\partial P}{\partial x} \cdot r^2 + c \rightarrow ①$

By Applying Boundary conditions, $r=R, u=0$

$$0 = \frac{1}{4\mu} \frac{\partial P}{\partial x} R^2 + c$$

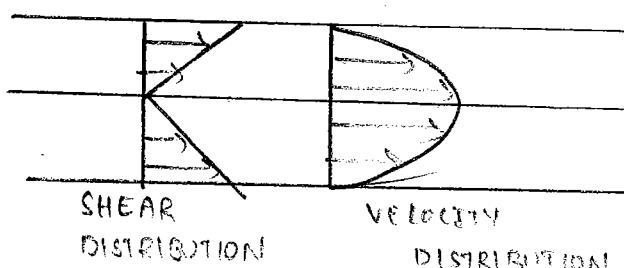
$$c = -\frac{1}{4\mu} \frac{\partial P}{\partial x} R^2$$

From ① $\Rightarrow u = \frac{1}{4\mu} \frac{\partial P}{\partial x} r^2 - \frac{1}{4\mu} \frac{\partial P}{\partial x} R^2$

$$u = -\frac{1}{4\mu} \frac{\partial P}{\partial x} (R^2 - r^2)$$

$u, \frac{\partial P}{\partial x}$ & R are constant and velocity u varies with R Square of

This equation implies a parabola



SHEAR STRESS DISTRIBUTION IN TURBULENT FLOW :

Reynolds in 1886 developed an expression for Turbulent shear stress b/w two layers of a fluid at small distance apart

$$\tau = \rho u'v'$$

Where u' & v' = Fluctuating component of velocity in the direction of x & y due to Turbulence. As u' & v' changes, τ also changes. Hence to find shear stress time average on both sides of eq

$$\bar{\tau} = \overline{\rho u' v'}$$

This turbulent stress is called "Reynold stress."

Turbulent Flow Of Velocity Distribution :—

In Pipes :—

In case of Turbulent flow, the total shear stress at any point is the sum of viscous shear stress & Turbulent shear stress. Viscous shear stress is negligible except near the Boundary.

$$\therefore u = \frac{u_*}{k} \log_e y + c \rightarrow ①$$

u_* → shear velocity

u → velocity distribution

k → Karman constant ($k=0.4$)

By applying Boundary conditions,

$$y=R, u=u_{\max}$$

$$u_{\max} = \frac{u_*}{k} \log_e R + c$$

(4)

$$C = U_{max} - \frac{U_*}{K} \log_e R$$

$$\begin{aligned} \text{From (1)} \Rightarrow u &= \frac{U_*}{K} \log_e y + U_{max} - \frac{U_*}{K} \log_e R \\ &= U_{max} + \frac{U_*}{K} (\log_e y - \log_e R) \\ &= U_{max} + \frac{U_*}{0.4} \log_e \left(\frac{y}{R} \right) \quad [\because K=4] \\ \therefore \boxed{u = U_{max} + 2.5 U_* \log_e \left(\frac{y}{R} \right)} \end{aligned}$$

this equation is Applicable to both Rough & smooth pipes.

also written as, $U_{max} - u = -2.5 U_* \log_e \left(\frac{y}{R} \right)$

$$U_{max} - u = 2.5 U_* \log \left(\frac{R}{y} \right)$$

÷ by ' U_* ' we get

$$\begin{aligned} \frac{U_{max} - u}{U_*} &= 2.5 \log \left(\frac{R}{y} \right) \\ &= 2.5 \times 2.3 \log_{10} \left(\frac{R}{y} \right) \end{aligned}$$

$$\left[\because \log_e \left(\frac{R}{y} \right) = 2.3 \times \log_{10} \left(\frac{R}{y} \right) \right]$$

$$\boxed{\frac{U_{max} - u}{U_*} = 5.75 \log_{10} \left(\frac{R}{y} \right)}$$

$U_{max} - u \rightarrow$ called
velocity Defect

Problem :-

- A pipe-line carrying water has average height of irregularities projecting from the surface of the boundary of the pipe as 0.15 mm. What type of boundary it is? The shear stress developed is 4.9 N/m². The kinematic viscosity of water is 0.1 stokes

AI-

Average Height of Irregularities $K = 0.15 \text{ mm}$

$$= 0.15 \times 10^{-3} \text{ m}$$

shear stress $\tau_0 = 4.9 \text{ N/m}^2$

Kinematic viscosity $\nu = 0.01$ stokes $= 0.1 \times 10^{-4} \text{ m}^2/\text{s}$

$$\rho = 1000 \text{ kg/m}^3$$

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07 \text{ m/s}$$

$$\text{Roughness Reynold number} = \frac{u_* k}{\nu} = \frac{0.07 \times 0.15 \times 10^{-3}}{0.1 \times 10^{-4}} = 10.5$$

$\frac{u_* k}{\nu}$ lies b/w 4 & 100.

Note :

Velocity Distribution for Turbulent flow in Rough pipes

$$\frac{u}{u_*} = 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5$$

Hagen Poiseuille Formula :-

Already we proved in velocity distribution of Laminar flow

$$u = -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2]$$

$$\text{When } r=0, u=u_{\max} \text{ then } u_{\max} = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 \rightarrow ①$$

The average velocity \bar{u} is obtained by dividing discharge of fluid across the section by the area of the pipe. The Discharge is obtained by considering flow through a circular ring element of radius r & thickness dr

$$dQ = \text{Velocity at a radius } r * \text{Area of ring element} \\ = u_* 2\pi r dr$$

$$= -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2] \times 2\pi r dr$$

(5)

$$\begin{aligned}
 Q &= \int_0^R -\frac{1}{4\mu} \frac{\partial P}{\partial x} (R^2 - r^2) \times 2\pi r dr \\
 &= -\frac{1}{4\mu} \left(\frac{\partial P}{\partial x} \right) \times 2\pi \int_0^R (R^2 - r^2) r dr \\
 &= -\frac{1}{4\mu} \frac{\partial P}{\partial x} \times 2\pi \int_0^R (R^2 r - r^3) dr \\
 &= \frac{1}{4\mu} \left(-\frac{\partial P}{\partial x} \right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4} \right]_0^R \\
 &= \frac{1}{4\mu} \left(-\frac{\partial P}{\partial x} \right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4} \right] \\
 Q &= \frac{\pi}{8\mu} \left(-\frac{\partial P}{\partial x} \right) R^4
 \end{aligned}$$

Average velocity $\bar{u} = \frac{Q}{\text{Area}}$

$$\bar{u} = \frac{\frac{\pi}{8\mu} \left(-\frac{\partial P}{\partial x} \right) R^4}{\pi R^2}$$

$$\therefore \bar{u} = \frac{1}{8\mu} \left(-\frac{\partial P}{\partial x} \right) R^2 \rightarrow ②$$

$$\frac{①}{②} \Rightarrow \frac{U_{\max}}{\bar{u}} = \frac{-\frac{1}{4\mu} \frac{\partial P}{\partial x} R^2}{-\frac{1}{8\mu} \frac{\partial P}{\partial x} R^2}$$

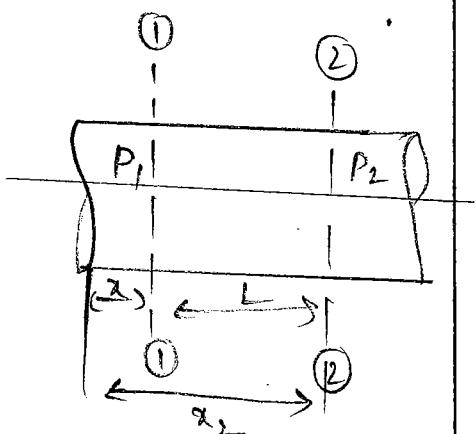
$$\therefore \frac{U_{\max}}{\bar{u}} = 2.$$

From ② $\Rightarrow -\frac{\partial P}{\partial x} = \frac{8\mu \bar{u}}{R^2}$

Integrating above w.r.t x , we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu \bar{u}}{R^2} dx$$

$$-[P_1 - P_2] = \frac{8\mu \bar{u}}{R^2} [x_1 - x_2]$$



$$P_1 - P_2 = \frac{8\mu \bar{u}}{R^2} [x_2 - x_1] \quad \left[\text{from dig } x_2 - x_1 = L \right]$$

$$\begin{aligned} P_1 - P_2 &= \frac{8\mu \bar{u}}{R^2} (L) \\ &= \frac{8\mu \bar{u}}{\left(\frac{D}{2}\right)^2} (L) \end{aligned}$$

$$\therefore P_1 - P_2 = \frac{32\mu \bar{u}}{D^2} \cdot L \quad P_1 - P_2 \rightarrow \text{Drop pressure}$$

$$\text{Loss of Head} = \frac{P_1 - P_2}{\rho g}$$

$$\therefore \frac{P_1 - P_2}{\rho g} = h_f = \frac{32\mu \bar{u} L}{\rho g D^2}$$

\therefore this eq is called Hagen poiseuille formula

1. A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m. calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank of 30 sec ?

A1

$$\mu = 0.97 \text{ poise} = \frac{0.97}{10} = 0.097 \text{ Ns/m}^2$$

$$\text{Relative density} = 0.9$$

$$\rho = 0.9 \times 1000 = 900 \text{ kg/m}^3$$

$$\text{Dia of pipe, } D = 100 \text{ mm} = 0.1 \text{ m}$$

$$L = 10 \text{ m}$$

$$\text{Mass of Oil collected, } M = 100 \text{ kg}$$

$$t = 30 \text{ sec}$$

$$P_1 - P_2 = \frac{32\mu \bar{u} L}{D^2}$$

(6)

$$\bar{u} = \text{Avg velocity} = \frac{Q}{\text{Area}}$$

$$\text{mass} = \frac{100}{30} \text{ kg/s}$$

$$= P_0 \times Q$$

$$\frac{100}{30} = 900 \times Q$$

$$\therefore Q = 0.0037 \text{ m}^3/\text{s}$$

$$\bar{u} = \frac{Q}{A} = \frac{0.0037}{\frac{\pi D^2}{4}} = 0.491 \text{ m/s } (\because D = 0.1)$$

For viscous flow $Re < 2000$

$$\therefore Re^* = \frac{\rho V D}{\mu} \quad V = \bar{u}$$

$$Re = \frac{900 \times 0.491 \times 0.1}{0.097}$$

$$= 436.91 (< 2000)$$

$$\therefore P_1 - P_2 = \frac{32 \mu \bar{u} L}{D^2}$$

$$= \frac{32 \times 0.097 \times 0.491 \times 10}{(0.1)^2}$$

$$\underline{\underline{P_1 - P_2 = 0.1462 \text{ N/cm}^2}}$$

Loss of Head due to Friction in Viscous flow in relation with Reynolds number :-

From Hagen Poiseuille formula, $h_f = \frac{32 \mu \bar{u} L}{\rho g D^2} \rightarrow ①$

Loss of Head due to friction is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{D \times g} = \frac{4 \cdot f \cdot L \cdot \bar{u}^2}{D \times g} \rightarrow ② \quad \because V = \bar{u}$$

$$① = ②,$$

$f \rightarrow$ coefficient of
Friction b/w

Pipe & fluid

$$\frac{32 \mu \bar{u} L}{\rho g D^2} = \frac{4 \cdot f \cdot L \cdot \bar{u}^2}{D \times g}$$

$$f = \frac{16H}{P.D.U}$$

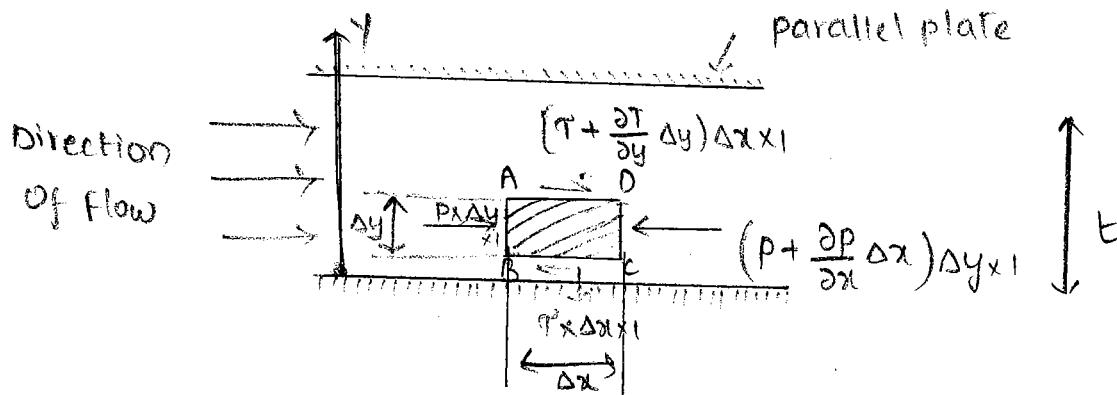
$$F = 16 \times \frac{H}{P.V.D} \quad (\because U=v)$$

$$= 16 \times \frac{1}{\left(\frac{P.V.D}{H}\right)}$$

$$\boxed{F = \frac{16}{Re}}$$

$$(\because Re = \frac{P.V.D}{H})$$

Flow of viscous flow between two parallel plates :-



Consider a fluid element of length ' Δx ' & thickness ' Δy ' at a distance ' y ' from lower fixed plate. If P is Intensity of pressure on AB & $(P + \frac{\partial P}{\partial x} \Delta x)$ on CD. let τ is shear stress on BC and $(\tau + \frac{\partial \tau}{\partial y} \Delta y)$. on AD

1. Pressure force , $P \times \Delta y \times 1$ on AB

2. Pressure force $(P + \frac{\partial P}{\partial x} \Delta x) \Delta y \times 1$ on CD

3. shear force $\tau \times \Delta x \times 1$ on BC

4. shear force $(\tau + \frac{\partial \tau}{\partial y} \Delta y) \Delta x \times 1$ on AD

there is no acceleration and hence resultant direction flow is zero.

$$P \times \Delta y \times 1 - (P + \frac{\partial P}{\partial x} \Delta x) \Delta y \times 1 - \tau \times \Delta x \times 1 + (\tau + \frac{\partial \tau}{\partial y} \Delta y) \Delta x \times 1 = 0$$

$$-\frac{\partial P}{\partial x} \Delta x \Delta y + \frac{\partial \tau}{\partial y} \cdot \Delta y \Delta x = 0$$

(7)

$$\div \text{ by } \Delta x \Delta y, \quad \frac{\partial p}{\partial x} = \frac{\partial T}{\partial y}$$

i) Velocity Distribution:

$$\text{From } \tau = \mu \frac{du}{dy}$$

$$\begin{aligned} \frac{\partial p}{\partial x} &= \frac{\partial}{\partial y} \left(\mu \frac{du}{dy} \right) \quad \text{from } \left(\frac{\partial p}{\partial x} = \frac{\partial T}{\partial y} \right) \\ &= \mu \frac{\partial^2 u}{\partial y^2} \end{aligned}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{\partial p}{\partial x}$$

Integrating above eq w.r.t y'

$$\frac{\partial u}{\partial y} = \frac{1}{\mu} \frac{\partial p}{\partial x} y + c_1$$

$$\text{Again, } u = \frac{1}{\mu} \frac{\partial p}{\partial x} \frac{y^2}{2} + c_1 y + c_2 \rightarrow (1) \quad (\because \frac{\partial p}{\partial x} = \text{constant})$$

By applying Boundary conditions,

$$\text{At } y=0, u=0 \Rightarrow 0 = \frac{1}{\mu} \frac{\partial p}{\partial x} \times 0 + c_1 \times 0 + c_2$$

$$c_2 = 0$$

$$\text{At } y=t, u=0 \Rightarrow 0 = \frac{1}{\mu} \frac{\partial p}{\partial x} \frac{t^2}{2} + c_1 t + 0 \quad (\because c_2 = 0)$$

$$\therefore c_1 = -\frac{1}{2\mu} \frac{\partial p}{\partial x} t$$

$$\text{From (1)} \Rightarrow u = \frac{1}{2\mu} \frac{\partial p}{\partial x} y^2 + y \left(-\frac{1}{2\mu} \frac{\partial p}{\partial x} t \right)$$

$$\boxed{\therefore u = -\frac{1}{2\mu} \frac{\partial p}{\partial x} [ty - y^2]}$$

ii) Ratio of maximum velocity to Average velocity:

$$\text{When } y = \frac{t}{2}, \quad u = -\frac{1}{2\mu} \frac{\partial p}{\partial x} \left[t \times \frac{t}{2} - \left(\frac{t}{2} \right)^2 \right]$$

$$\therefore u_{max} = -\frac{1}{8\mu} \frac{\partial p}{\partial x} t^2 \rightarrow ②$$

We know discharge = velocity * Area

dQ = velocity at distance y * Area of strip

$$= -\frac{1}{2\mu} \frac{\partial p}{\partial x} [ty - y^2] dy *$$

$$Q = \int_0^t dQ = \int_0^t -\frac{1}{2\mu} \frac{\partial p}{\partial x} [ty - y^2] dy$$

$$= -\frac{1}{2\mu} \frac{\partial p}{\partial x} \left[\frac{ty^2}{2} - \frac{y^3}{3} \right]_0^t$$

$$= -\frac{1}{12\mu} \frac{\partial p}{\partial x} t^3$$

$$\bar{u} = \frac{Q}{\text{Area}} = \frac{-\frac{1}{12\mu} \frac{\partial p}{\partial x} t^3}{tx} = -\frac{1}{12\mu} \frac{\partial p}{\partial x} t^2 \rightarrow ③$$

$$\frac{②}{③} \Rightarrow \frac{u_{max}}{\bar{u}} = \frac{-\frac{1}{8\mu} \frac{\partial p}{\partial x} t^2}{-\frac{1}{12\mu} \frac{\partial p}{\partial x} t^2}$$

$$\boxed{\frac{u_{max}}{\bar{u}} = \frac{3}{2}}$$

Drop of pressure head for given length :-

$$③ \Rightarrow \bar{u} = -\frac{1}{12\mu} \frac{\partial p}{\partial x} t^2$$

$$\frac{\partial p}{\partial x} = -\frac{12\mu \bar{u}}{t^2}$$

Integrating w.r.t 'x'

$$\int_2^1 dp = \int_2^1 -\frac{12\mu \bar{u}}{t^2} dx$$

$$P_1 - P_2 = -\frac{12\mu \bar{u}}{t^2} (x_1 - x_2)$$

$$P_1 - P_2 = \frac{12\mu \bar{u}}{t^2} (x_2 - x_1)$$

$$P_1 - P_2 = \frac{12\mu UL}{L^2} \quad \therefore x_2 - x_1 = L$$

$$h_f = \frac{P_1 - P_2}{\rho g} = \frac{12\mu UL}{\rho g L^2}$$

iv) Shear stress Distribution :-

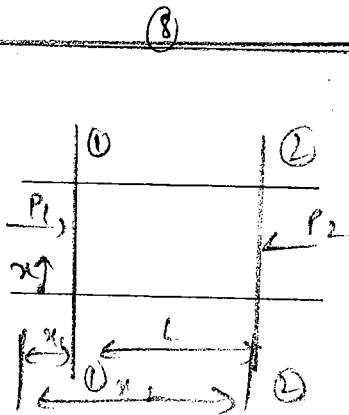
$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{\partial}{\partial y} \left[-\frac{1}{2\mu} \frac{\partial p}{\partial x} (ty - y^2) \right]$$

$$\tau = -\frac{1}{2} [t - 2y] \frac{\partial p}{\partial x}$$

By Apply Boundary conditions, $y=0$

$$\therefore \tau = -\frac{1}{2} \frac{\partial p}{\partial x} \cdot t$$



1. Calculate i) pressure gradient along flow ii) the average velocity
 iii) the discharge for oil of viscosity 0.02 Ns/m^2 flowing b/w two stationary parallel plates 1 m wide maintained 10 mm apart. The velocity midway b/w plates is 2 m/s,

viscosity $\mu = 0.2 \text{ N.s/m}^2$, $b = 1 \text{ m}$, $t = 10 \text{ mm} = 0.1 \text{ m}$

$$U_{max} = 2 \text{ m/s}$$

i) Pressure gradient

$$U_{max} = -\frac{1}{8\mu} \frac{\partial p}{\partial x} \cdot t^2$$

$$2 = -\frac{1}{8 \times 0.2} \frac{\partial p}{\partial x} (0.1)^2$$

$$\frac{\partial p}{\partial x} = -3200 \text{ N/m}^2$$

ii) $\frac{U_{max}}{\bar{U}} = \frac{3}{2}$

$$\therefore \bar{U} = \frac{2}{3} \times U_{max}$$

$$= \frac{2}{3} \times 2$$

$$\therefore \bar{U} = 1.33 \text{ m/s}$$

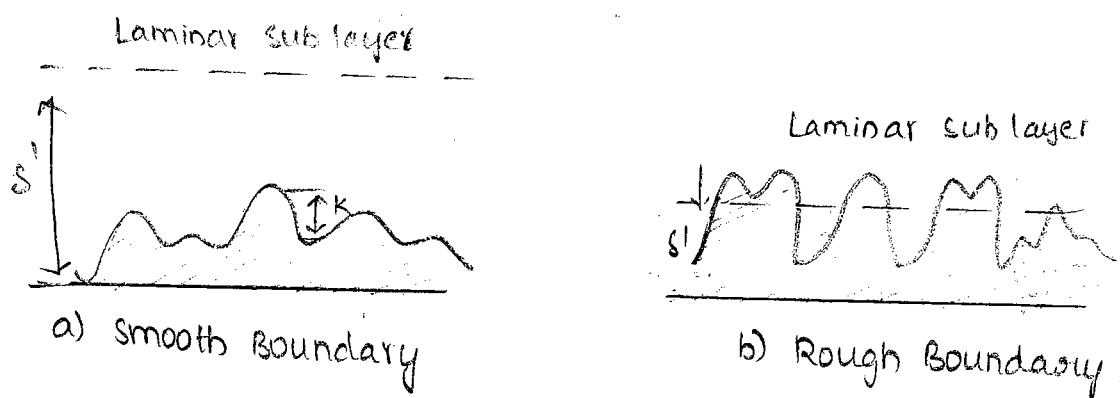
iii) $Q = \bar{U} \times \text{Area} = b \times t \times \bar{U} = 1 \times 0.1 \times 1.33$

$$Q = 0.0133 \text{ m}^3/\text{s}$$

Hydrodynamically Smooth & Rough Pipes :-

let K is the average height of irregularities projecting from the surface of a boundary

If value of K is large then Boundary is called ^{Rough} ~~rough~~ Boundary
If value of K is small then Boundary is called smooth Boundary
classification is based on Boundary characteristics



Viscous shear force predominates while the shear stress due to turbulence is negligible. This portion is known as Laminar sub layer. When shear stress due to turbulence are large as compared to viscous stress is known as turbulent zone.

The Eddies present in turbulent flow try to penetrate the laminar sub layer and reach surface of Boundary but due to great thickness of laminar sub layer the eddies are unable to reach the surface irregularities. This Boundary is called "hydrodynamically smooth Boundary." In this Reynolds number is much less as shown in fig (a)

If the Reynolds number of flow is increased, then thickness of laminar sub layer will decrease. The thickness of laminar sub layer becomes much smaller than average height ' K ' boundary acts as rough Boundary and irregularities of surface

are become above the laminar sublayer and eddies present in turbulent zone will come in contact with surface & lot of energy will be lost. This is called Hydrodynamically Rough Boundary

→ If $\frac{K}{\delta_1}$ is less than 0.25 → smooth

→ If $\frac{K}{\delta_1}$ is greater than 6.0 → Rough

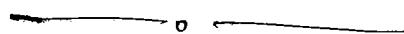
→ $0.25 < \frac{K}{\delta_1} < 6$. → Transition

* Roughness Reynolds Number = $\frac{U_* K}{V}$

→ If $\frac{U_* K}{V} < 4$ → smooth

→ $\frac{U_* K}{V}$ lies b/w 4 & 100 → Transition stage

→ $\frac{U_* K}{V} > 100$ → Rough





Darcy - Weisbach Equation : —

Consider uniform horizontal pipe, having steady flow as shown in fig. let 1-1 & 2-2 are two sections of pipe

let P_1 = pressure intensity at section 1-1

V_1 = velocity of flow at section 1-1

L = length of pipe between sections 1-1 & 2-2

d = diameter of pipe

f' = frictional resistance per unit wetted area per unit velocity

P_2, V_2 = pressure intensity & velocity of flow at 2-2

Applying Bernoulli's Eq's b/w 1-1 & 2-2,

Total head at 1-1 = Total head at 2-2 + Loss of Head due to

Friction b/w 1-1 & 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But

$z_1 = z_2$ as pipe is horizontal

$V_1 = V_2$ as dia of pipe is same at both sections

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f \quad (1) \quad h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \rightarrow (1)$$

Intensity of pressure will be reduced in the direction of flow by frictional resistance

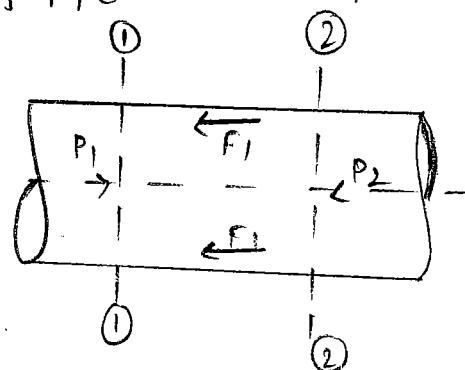
$$F_f = f' \times \pi d L \times V^2$$

Frictional Resistance = Frictional resistance per unit wetted area per unit velocity * wetted area * (velocity)²

\therefore wetted area = $\pi d \times L$

Velocity $V = V_1 = V_2$

πd = Perimeter = p



The forces acting on the fluid between sections 1-1 & 2-2 are:

1. Pressure force at section 1-1 = $P_1 A$, $A \rightarrow$ Area of pipe
2. Pressure force at section 2-2 = $P_2 A$
3. Frictional force F_f

Resolving all forces in horizontal direction,

$$P_1 A - P_2 A - F_f = 0$$

$$(P_1 - P_2)A = F_f = f' \times \rho \times L \times V^2$$

$$P_1 - P_2 = \frac{f' \times \rho \times L \times V^2}{A} \quad \because F_f = f' \rho L V^2$$

$$\text{From } (1) \Rightarrow P_1 - P_2 = \rho g h_f$$

$$\rho g h_f = \frac{f' \times \rho \times L \times V^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \rightarrow (3)$$

$$\text{In Eq (3), } \frac{P}{A} = \frac{\text{Wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2$$

$$\therefore h_f = \frac{f'}{\rho g} \cdot \frac{4 L V^2}{d} \rightarrow (4)$$

$$\frac{f'}{f} = \frac{f}{2}, \quad f \rightarrow \text{coefficient of friction}$$

$$(4) \text{ as, } h_f = \frac{f}{2g} \cdot \frac{4 L V^2}{d}$$

$$\boxed{\therefore h_f = \frac{4 f L V^2}{2 g d}}$$

This eq is called Barcy-weisbach Equation

Sometimes eq is written as

$$\boxed{h_f = \frac{f \cdot L \cdot V^2}{2 g \times d}}$$

(9)

Chezy's formula for loss of head due to friction in pipes :—

$$\text{We know loss of head } h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times v^2$$

$h_f \rightarrow$ loss of head due to friction, $P = \text{Wetted perimeter of pipe}$

$A \rightarrow$ Area of cross section of pipe, $L = \text{length of pipe}$

$v \rightarrow$ Mean velocity of flow.

The ratio of $\frac{A}{P} = \frac{\text{Area of flow}}{\text{Wetted perimeter}}$ is called Hydraulic mean

depth or hydraulic radius and is denoted by m .

$$\therefore \text{Hydraulic mean depth, } m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$$

$$\frac{A}{P} = \frac{m}{1} \quad (\text{or}) \quad \frac{P}{A} = \frac{1}{m}$$

$$h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L \times v^2 \Rightarrow v^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L}$$

$$v = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} \Rightarrow v = \sqrt{\frac{\rho g}{f'}} \sqrt{m \cdot \frac{h_f}{L}} \rightarrow ①$$

let $\sqrt{\frac{\rho g}{f'}} = c$, Where $c \rightarrow$ Chezy's constant

$\frac{h_f}{L} = i$, $i \rightarrow$ loss of Head per unit length of pipe

By substituting in ①, $v = c \sqrt{mi}$

This Eq is called Chezy's formula.

m is always equal to $\frac{d}{4}$

1. Find the head lost due to friction in a pipe of diameter 300 mm & length 50 m through which water is flowing at a velocity of 3 m/s using i) Darcy formula ii) Chezy formula, $c=60$

Take v for water = 0.01 stoke

Dia of Pipe $d = 300 \text{ mm} = 0.30 \text{ m}$

Length of Pipe $L = 50 \text{ m}$

Velocity = 3 m/s, C = 60,

Kinematic viscosity $\nu = 0.01 \times 10^{-4} \text{ m}^2/\text{s}$

i) Darcy formula $h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times g}$

f = coefficient of friction is a function of Reynolds number

$$Re = \frac{V \times d}{\nu} = \frac{3 \times 0.30}{0.01 \times 10^{-4}} = 9 \times 10^5$$

$$\therefore f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256$$

Head lost, $h_f = \frac{4 \times 0.00256 \times 50 \times 3^2}{0.3 \times 2 \times 9.81}$

$$h_f = 0.7828 \text{ m}$$

ii) chezy formula $V = C \sqrt{hi}$

$$C = 60, m = \frac{d}{4} = \frac{0.30}{4} = 0.075 \text{ m}$$

$$3 = 60 \sqrt{0.075 \times i} \Rightarrow i = 0.0333$$

$$i = \frac{h_f}{L} \Rightarrow \frac{h_f}{50} = 0.0333$$

$$h_f = 1.665 \text{ m}$$

Note for Darcy Eq :

f = coefficient of friction which is a function of Reynolds number

$$f = \frac{16}{Re} \quad \text{for } Re < 2000$$

$$f = \frac{0.079}{(Re)^{1/4}} \quad \text{Re varying from 4000 to } 10^6$$

Minor Energy Losses :-

The loss of Head (or) energy due to friction in a pipe is known as Major Loss while the loss of energy due to change of velocity of flowing fluid in magnitude (or) direction is called Minor Loss of Energy. These includes following cases :

1. Loss of Head due to sudden Enlargement
2. Loss of Head due to sudden contraction
3. Loss of Head at Entrance of a pipe
4. Loss of Head at Exit of a pipe
5. Loss of Head due to an obstruction in a pipe
6. Loss of Head due to bend in the pipe
7. Loss of Head in various pipe fittings

Loss of Head due to sudden Enlargement :-

Consider a liquid flowing through a pipe which has sudden Enlargement. Consider two sections before and after enlargement

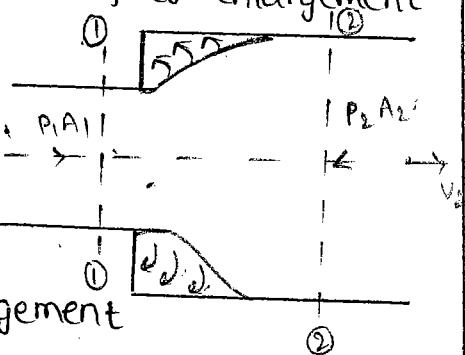
P_1, P_2 = Pressure intensities at ①-① & 2-2

V_1, V_2 = Velocity of flows at 1-1 & 2-2

A_1, A_2 = Area of pipes at 1-1 & 2-2

h_e = Loss of Head due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$



Loss of Head due to sudden contraction :-

Consider a liquid flowing in a pipe which has a sudden contraction in area as shown. Consider two sections before and after contraction. As the liquid flows from larger pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at section c-c.

This section c-c is known as Vena-Contracta. After section c-c sudden enlargement takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena-contracta to smaller pipe

$$A_c = \text{Area of flow at } c-c$$

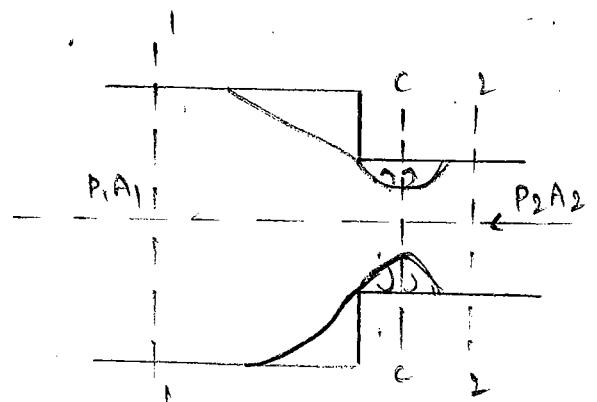
$$V_c = \text{Velocity of flow at } c-c$$

$$A_2 = \text{Area of flow at } 2-2$$

$$V_2 = \text{Velocity of flow at } 2-2$$

$$h_c = \text{Loss of Head due to sudden contraction}$$

$$h_c = 0.5 \frac{V_c^2}{2g}$$



Velocity loss of Head at Entrance of Pipe :-

This is loss of Energy which occurs when a liquid enters a pipe which is connected to a large tank (A) reservoir. This loss is similar to the loss of Head due to sudden contraction. This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded (B) bell mouthed entrance. In practice the value of loss of Head at entrance of a pipe with sharp cornered entrance is taken

$$= 0.5 \frac{V^2}{2g}$$

$$h_i = 0.5 \frac{V^2}{2g}$$

Loss of Head at Exit of a Pipe :-

This is loss of Head due to the Velocity of liquid at outlet of pipe which is dissipated in the form of free jet or it is lost in the tank or reservoir. This loss is equal to $\frac{V^2}{2g}$, it is denoted by h_o

$$h_o = \frac{V^2}{2g}$$

(4)

Loss of Head due to Obstruction in a pipe :-

Whenever there is an obstruction in a pipe, the loss of energy takes place due to the reduction of the area of cross section of the pipe at place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown.

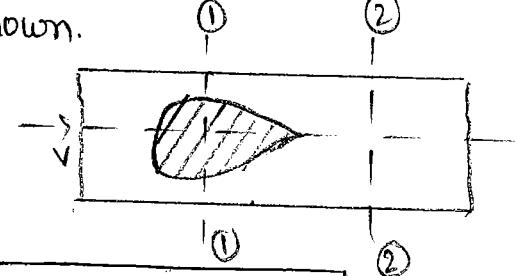
$$a = \text{max area of obstruction}$$

$$A = \text{Area of pipe}$$

$$V = \text{Velocity of liquid in pipe}$$

$$\text{Head loss due to obstruction} =$$

$$C_c = \text{coefficient of contraction}$$



$$\frac{V^2}{2g} \left(\frac{A}{C_c(A-a)} - 1 \right)^2$$

Loss of Head due to Bend in pipe :-

When there is any bend in a pipe, the velocity of flow changes due to which the separation of flow from the boundary and also formation of eddies takes place. Thus energy is lost

$$h_b = \frac{Kv^2}{2g}$$

$h_b \rightarrow$ loss of head due to bend

$v \rightarrow$ velocity of flow

$K \rightarrow$ coefficient of bend

K depends on

Angle of Bend, Radius of curvature & Diameter of pipe

Loss of Head in various Pipe fittings :-

The loss of Head in the various pipe fittings such as valves, couplings etc... is expressed as

$$\frac{Kv^2}{2g}$$

$K \rightarrow$ coefficient of pipe fitting

$v \rightarrow$ velocity of flow

1. The difference in water surface levels in two tanks which are connected by three pipes in series of length 300 m, 170 m & 210 m and of diameter 300 mm, 200 mm & 400 mm respectively is 18 m. Determine the state of flow of water if co-efficient of friction are 0.005, 0.0052 & 0.0048 respectively, considering
 i) Minor losses also & ii) Neglecting minor losses

Al-

Given, Difference $H = 18 \text{ m}$

$$L_1 = 300 \text{ m} \quad L_2 = 170 \text{ m}, \quad L_3 = 210 \text{ m}$$

$$D_1 = 300 \text{ mm} \quad D_2 = 200 \text{ mm} \quad D_3 = 400 \text{ mm} \\ = 0.3 \text{ m} \quad = 0.2 \text{ m} \quad = 0.4 \text{ m}$$

$$f_1 = 0.005, \quad f_2 = 0.0052, \quad f_3 = 0.0048$$

i) Considering minor losses

v_1, v_2 & v_3 are velocities of pipes 1, 2 & 3

From continuity eq, $A_1 v_1 = A_2 v_2 = A_3 v_3$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2} \times v_1$$

$$\therefore v_2 = 2.225 v_1$$

$$v_3 = \frac{A_2 v_1}{A_3} = \frac{d_1^2}{d_3^2} \times v_1$$

$$\therefore v_3 = 0.5625 v_1$$

We have Eq for minor losses when they are considered

$$H = \frac{0.5 v_1^2}{2g} + \frac{4f_1 L_1 v_1^2}{2g \times d_1} + \frac{0.5 v_2^2}{2g} + \frac{4f_2 L_2 v_2^2}{2g \times d_2} + \frac{(v_2 - v_3)^2}{2g} + \frac{4f_3 L_3 v_3^2}{d_3 \times 2g} + \frac{v_3^2}{2g}$$

Substituting v_2 & v_3 :

$$18 = \frac{0.5 v_1^2}{2g} + \frac{4 \times 0.005 \times 300 \times v_1^2}{2 \times g \times 0.3} + \frac{0.5 \times (2.225 v_1^2)^2}{2g} +$$

(5)

$$+ 4 \times 0.0052 \times A_0 \times \frac{2.25(v_1^2)}{0.2 \times 2g} + \frac{(2.25v_1 - 0.562v_1)^2}{2g} + \frac{4 \times 0.0048 \times 210 \times (0.5625v_1)^2}{0.4 \times 2g}$$

$$+ \frac{(0.5625v_1)^2}{2g}$$

$$12 = \frac{v_1^2}{2g} [118.887] \Rightarrow v_1 = 1.407 \text{ m/s}$$

Rate of flow $Q = \text{Area} * \text{velocity}$

$$= \frac{\pi}{4} d^2 * v_1$$

$$= (0.3)^2 \frac{\pi}{4} * 1.407$$

$$\therefore Q = 99.45 \text{ liter/sec}$$

ii) Neglecting Minor Losses :

$$H = \frac{4f_1 L_1 v_1^2}{2g d_1} + \frac{4f_2 L_2 v_2^2}{2g d_2} + \frac{4f_3 L_3 v_3^2}{2g d_3}$$

$$12 = \frac{v_1^2}{2g} \left(\frac{4 \times 0.005 \times 300}{0.3} + \frac{4 \times 0.0052 \times 90 \times (2.25)^2}{0.2g} + \frac{4 \times 0.0048 \times 210 \times (0.5625)^2}{0.4} \right)$$

$$12 = \frac{v_1^2}{2g} [112.694] \Rightarrow v_1 = 1.445 \text{ m/sec}$$

Discharge $Q = v_1 * A_1$

$$= \frac{\pi}{4} (0.3)^2 * 1.445$$

$$\therefore Q = 102.1 \text{ liters/sec}$$

Flow through pipes in series (i) Flow through compound pipes

Pipes in series (i) compound pipes are defined as the pipes of different lengths & different diameters connected end to end to form a pipe line.

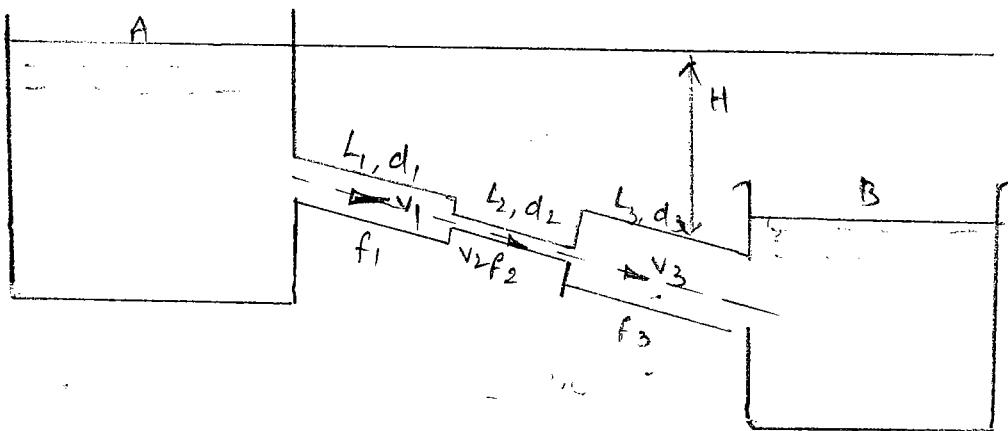
let l_1, l_2, l_3 = length of pipes 1, 2 & 3 respectively

d_1, d_2, d_3 = diameter of pipes 1, 2 & 3 respectively

v_1, v_2, v_3 = velocity of flow through pipes 1, 2, 3

f_1, f_2, f_3 = coefficients of friction for pipes 1, 2, 3

H = difference of water level in two tanks.



The discharge passing through each pipe is same

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

the difference in liquid surface levels is equal to the sum of total head loss in the pipes

$$H = \frac{0.5 V_1^2}{2g} + \frac{4f_1 L_1 V_1^2}{2g d_1} + \frac{0.5 V_2^2}{2g} + \frac{4f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3 L_3 V_3^2}{2g d_3} + \frac{V_3^2}{2g} \rightarrow ①$$

If mind losses are neglected then eq becomes

$$H = \frac{4f_1 L_1 V_1^2}{2g d_1} + \frac{4f_2 L_2 V_2^2}{2g d_2} + \frac{4f_3 L_3 V_3^2}{2g d_3}$$

If coefficient of friction is same for pipes

$$f_1 = f_2 = f_3 = f$$

$$H_f = \frac{4f L_1 V_1^2}{2g d_1} + \frac{4f L_2 V_2^2}{2g d_2} + \frac{4f L_3 V_3^2}{2g d_3}$$

$$H = \frac{4f}{2g} \left(\frac{L_1 V_1^2}{d_1} + \frac{L_2 V_2^2}{d_2} + \frac{L_3 V_3^2}{d_3} \right)$$

(6)

Flow through parallel plates : —

Consider a main pipe which divides into two (or) more branches as shown in fig and again join together downstream to form a single pipe then the Branch pipes are said to be connected in parallel. the discharge through the main is increased by connecting Pipes in parallel.

The rate of flow in main pipe is equal to the sum of rate of flow through branch pipes. We have

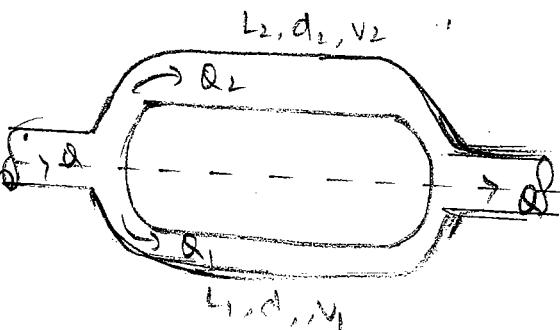
$$Q = Q_1 + Q_2$$

Loss of Head for branch pipe 1 = Loss of Head for branch pipe 2

$$\frac{Hf_1 L_1 V_1^2}{d_1 \times g} = \frac{Hf_2 L_2 V_2^2}{d_2 \times g}$$

$$\boxed{\frac{L_1 V_1^2}{d_1 \times g} = \frac{L_2 V_2^2}{d_2 \times g}}$$

$$\therefore f_1 = f_2$$



Energy :

- * The ability to do work
- * the ability to exert force on an object to move it.

Pressure Energy :

- It is a force that produce the flow in a pipe line
- It is expressed in terms of Head i.e in metres of liquid
- It is equal to the pressure (P) divided by specific wt of liquid
- Pressure head = $\frac{P}{W}$

Potential Energy :

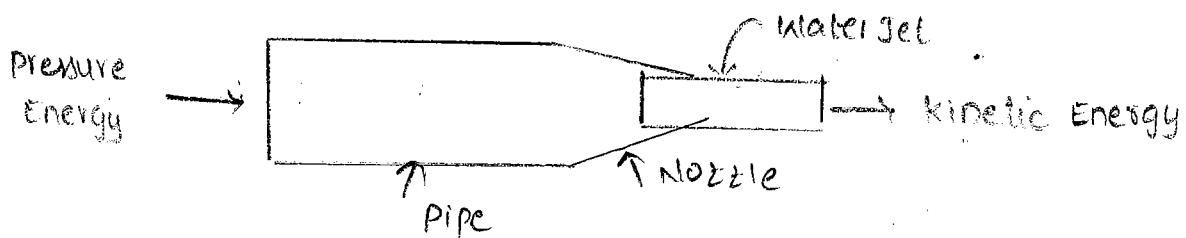
- * It is energy stored by liquid due to its elevated position
- * It is expressed in terms of Head i.e in meters
- * It is equal to the height of liquid from datum

* Potential Energy Head = 2 meters

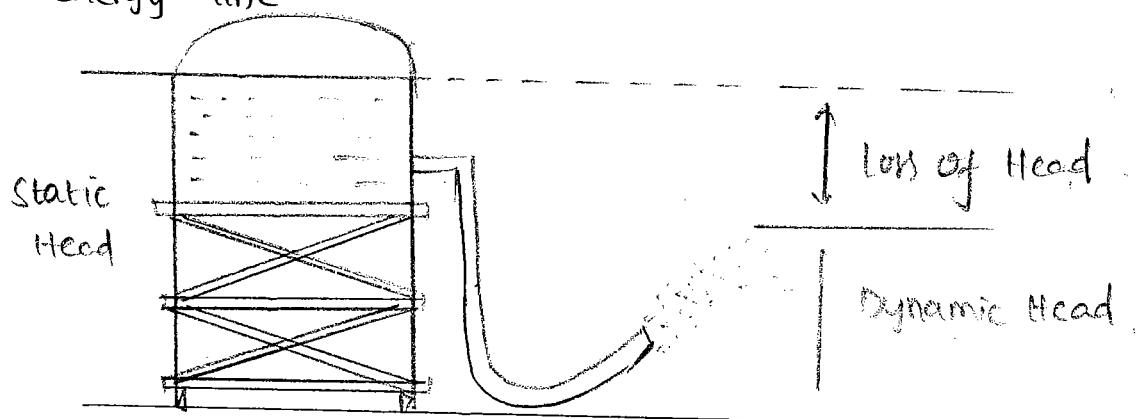
Kinetic Energy :-

- A moving liquid possess kinetic Energies
 - It is the energy possessed by the liquid due to its motion
 - It is expressed in terms of head of liquid i.e. $\frac{v^2}{2g}$ in meters
- Kinetic Energy Head = $\frac{v^2}{2g}$ meters

During the fluid flow the change of energy from one form to the other takes place.



this can be shown graphically by using Hydraulic gradient line and Total Energy line

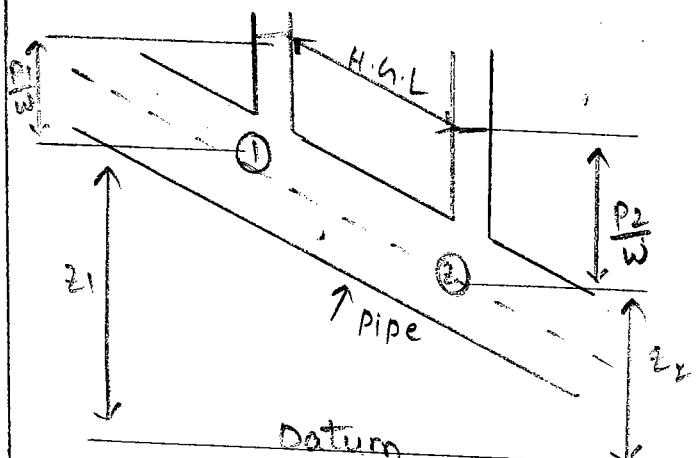


Hydraulic Gradient line :-

It is the line joining the sum of pressure heads & datum heads at different sections of the pipe.

$$\text{Piezometric Head at section } 1 = \frac{P_1}{w} + z_1$$

$$\text{Piezometric Head at section } 2 = \frac{P_2}{w} + z_2$$



HGL Joins $\frac{P_1}{w} + z_1$ with $\frac{P_2}{w} + z_2$ as shown in fig.

Total Energy line :-

It is the line joining the sum of Pressure Head, velocity Head & datum Head at different sections of the pipe.

$$\text{Total Energy at section 1} = \frac{P_1}{w} + \frac{V_1^2}{2g} + z_1$$

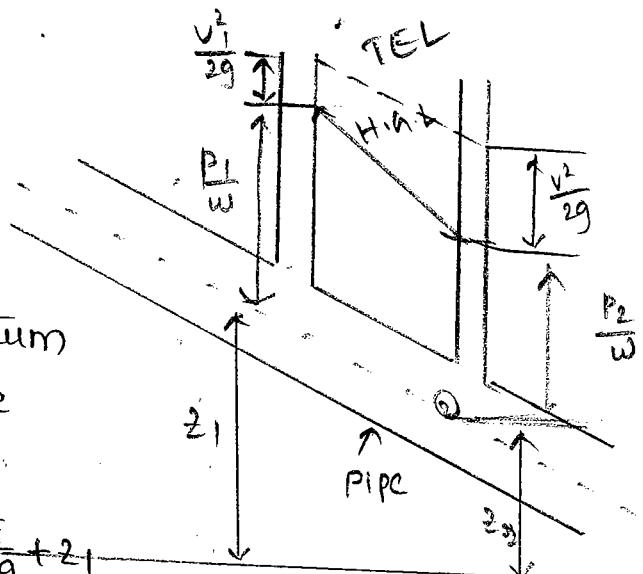
$$\text{Total Energy at section 2} = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

TEL Joins $\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1$ with $\frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$

Pipe Network :-

A Pipe network is an interconnected system of pipes forming several loops (or) circuits. The pipe examples of such networks of pipes are the municipal water distribution systems in cities and laboratory supply system. In such system, it is required to determine the distribution of flow through various pipes of network.

- The flow into each junction must be equal to the flow out of the junction. This is due to continuity Equation



ii) The algebraic sum of Head losses round each loop must be zero. This means that in each loop, the loss of head due to flow in clockwise direction must be equal to the loss of head due to flow in anticlockwise direction.

iii) The head loss in each pipe is expressed as $h_f = r\alpha^n$. The value of α depends on the length, diameter & co-efficient of friction of pipe.

Value of n for turbulent flow is "2".

$$\begin{aligned}
 h_f &= \frac{4 \times f \times L \times V^2}{D \times 2g} & \left(V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} D^2} \right) \\
 &= \frac{4 f L \times \left(\frac{Q}{A} \right)^2}{D \times 2g} \\
 &= \frac{4 f L \times Q^2}{D \times 2g \left(\frac{\pi}{4} D^2 \right)^2} \\
 &= \frac{4 f L Q^2}{2g \left(\frac{\pi}{4} \right)^2 D^5} \\
 h_f &= r \alpha^2
 \end{aligned}$$

$\therefore r = \frac{4 f L}{2g \left(\frac{\pi}{4} \right)^2 D^2}$

Hardy Cross Method :-

The procedure for Hardy cross method is as follows :

1. In this method a trial distribution of discharges is made arbitrary but in such a way that continuity Eq is satisfied at each junction.
2. With the assumed values of Q , the head loss in each pipe is calculated according to the equation $h_f = r\alpha^2$
3. Now consider any loop. The algebraic sum of head losses round each loop must be zero. This means that in each loop the loss of head due to flow in clockwise direction must be

(8)

equal to loss of Head due to flow in anticlockwise direction.

4. Now calculate the net head loss around each loop considering the head loss to be positive in clockwise flow and to be negative in anticlockwise flow.

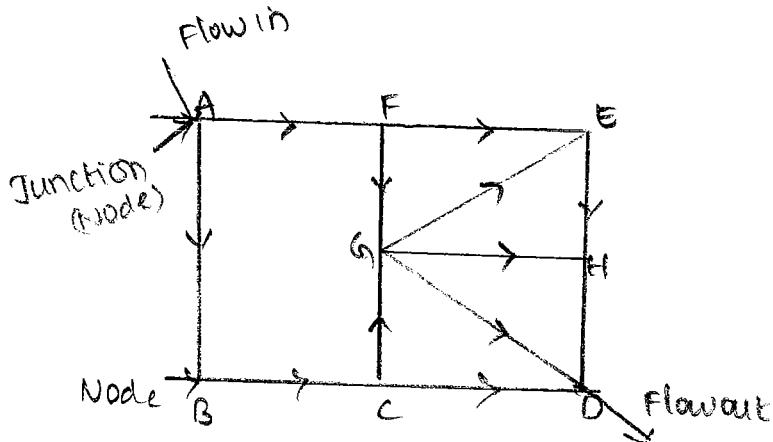
If the net head loss due to assumed Q values \rightarrow the loop is zero, then the assumed values of Q in that loop is correct. But if the net head loss due to assumed values of Q is not zero then the Assumed values of Q are corrected by introducing a correction ΔQ for the flows

$$\text{Correction factor } \Delta Q = \frac{-\sum r Q_0^n}{\sum r \cdot n \cdot Q_0^{n-1}}$$

5. If the value of ΔQ comes out to be positive, then it should be added to the flows in clockwise direction and subtracted from the flows in anticlockwise direction.

6. Some pipes may be common to two circuits, then the two corrections are applied to these pipes.

7. After the corrections have been applied to each pipe in a loop and to all loops, a second trial calculation is made for all loops. The procedure is repeated till ΔQ becomes negligible.



Loops : ABEGFA, FEGIF, GIHGI, GHHDG & GICDG



