

Lecture HandoutFLowsObjectives:

At the end of this topic, you will be able to:

- Describe Reynold's experiment
- Differentiate the types of flow
- Explain flow between parallel plates
- Detail on flow through long plates
- Describe flow through inclined plates
- Solve problems based on the aforesaid concepts.

Outcomes:

By the end of this topic, you will be able to.

- Understand Reynold's experiment
- Know about the types of flow
- Understand the flow between parallel plates
- Know about the flow through long plates and inclined plates.
- Illustrate problems based on the aforesaid concepts.

Introduction:

- A pipe is a closed conduit which is used for carrying fluid under pressure
- Pipes are commonly circular in section.
- As the pipes carry fluids under pressure, the pipes always run full.
- The fluid flowing in a pipe is always subjected to resistance due to shear forces.
- It occurs between fluid particles and the boundary walls of the pipe and between the fluid particles themselves resulting from the viscosity of the fluid.
- The resistance to the flow of fluid is generally known as frictional resistance.

→ Since certain amount of energy possessed by the flowing fluid will be consumed in overcoming this resistance to the flow, there will always be some loss of energy in the direction of flow.

→ Loss of energy depends on the type of flow. The flow of fluid in a pipe may be either laminar or turbulent.

→ The existence of the two types of flow, viz., Laminar and turbulent, was first demonstrated by Osborne Reynolds in 1883, with the help of a simple experiment discussed.

Reynold's Experiment :

→ The setup shown in figure consists of the following

- * Constant head tank filled with water
- * Small tank containing dye
- * Glass tube with a bell mouthed entrance
- * Regulating valve

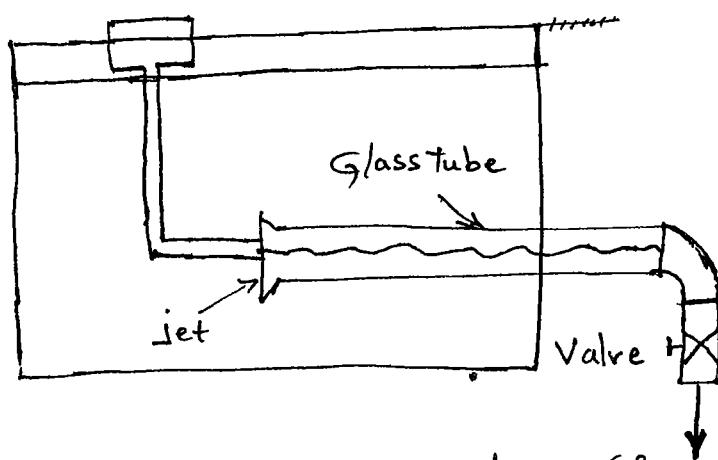


Fig. Reynold's apparatus for demonstrating the types of flow.

→ The water was made to flow from the tank through the glass tube into atmosphere and the velocity of flow, was varied by adjusting the regulating valve.

→ A liquid dye having the same specific weight as that of water, was introduced into the flow at the bell-mouth through a small tube.

Conclusions

→ From the experiments it was seen that when the velocity of flow was low.

→ The dye remained in the form of a straight and stable filament passing through the glass tube so steadily that it scarcely seemed to be in motion.

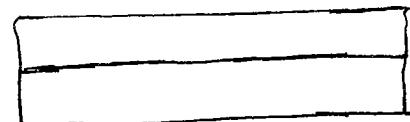
→ With increase in the velocity of flow a critical state was reached at which the filament of dye showed irregularities and began to waver.

→ With a further increase in the velocity of flow the fluctuations in the filament of dye became more intense.

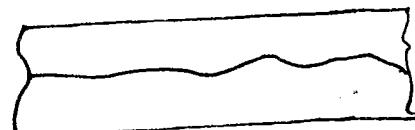
→ Ultimately the dye diffused over the entire cross section of the tube, due to the intermingling of the particles of the flowing fluid.

→ Figure shows the different states of the dye filament.

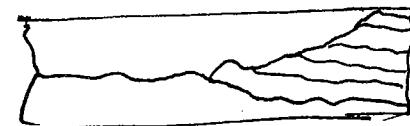
→ Reynolds deduced from his experiments that at low velocities the intermingling of the fluid particles was altogether absent and that the fluid particles moved in parallel layers of laminae, sliding past adjacent laminae but not mixing with them. This is the regime of Laminar flow (figure a)



(a) Laminar Flow



(b) Transition



(c) Turbulent Flow

Fig: Appearance of dye filament in glass tube

→ Since at higher velocities the dye filament diffused through the tube, it was apparent that the intermingling of the fluid particles was occurring, or in other words the flow was turbulent (figure(c)).

→ The velocity at which the flow changes from the laminar to turbulent for the case of a given fluid at a given temperature and in a given pipe is known as critical velocity.

→ The state of flow in between these two types of flow is known as 'transitional state' (or flow in transition).

On the basis of his experiments Reynolds discovered that the occurrence of a laminar and turbulent flow was governed by the relative magnitudes of the inertia and the viscous forces.

→ It was indicated by Reynolds that at low velocities of flow even for the fluids having very small viscosity, the viscous forces become predominant and therefore the flow is largely viscous in character.

→ However, at higher velocities of flow the inertial forces have predominance over the viscous forces. Reynolds related the inertia to viscous forces and arrived at a dimensionless parameter.

$$Re \text{ or } N_r = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v}$$

→ According to Newton's second law of motion the inertia force F_i is given by

$F_i = \text{mass} \times \text{acceleration}$

$= \rho \times \text{volume} \times \text{acceleration}$.

$$= \rho \times L^3 \times (V/T^2) = \rho L^2 V^2$$

Similarly viscous force F_v is given by Newton's law of viscosity as,

$$F_v = \tau \times \text{Area}$$

$$= \mu \frac{du}{dy} \times L^2 = (\mu VL)$$

$$Re \text{ or } N_r = \frac{(\rho L^2 V^2)}{\mu VL} = \frac{\rho VL}{\mu}$$

→ This dimensionless parameter is called Reynold's number, in which ρ and μ are respectively the mass density and viscosity of the flowing fluid, V is the characteristic (or representative) velocity of flow and L is the characteristic linear dimension.

→ In the case of flow through pipes the characteristic linear dimension L is taken as the diameter D of the pipe and the characteristic velocity is taken as the average velocity V of flow of fluid.

→ Thus Reynold's number becomes $\left(\frac{\rho DV}{\mu} \right)$ or $\left(\frac{VD}{\eta} \right)$ where $\left(\frac{\mu}{\rho} \right) = \eta$. Kinematic Viscosity of flowing fluid

→ The Reynold's number is therefore a very useful parameter in predicting whether the flow is laminar or turbulent.

- The limiting values of Reynolds number corresponding to which the flow of fluid in a pipe is either laminar or turbulent are discussed later.
- The existence of two flow regimes may also be indicated with the help of another simple experiment as shown in fig.

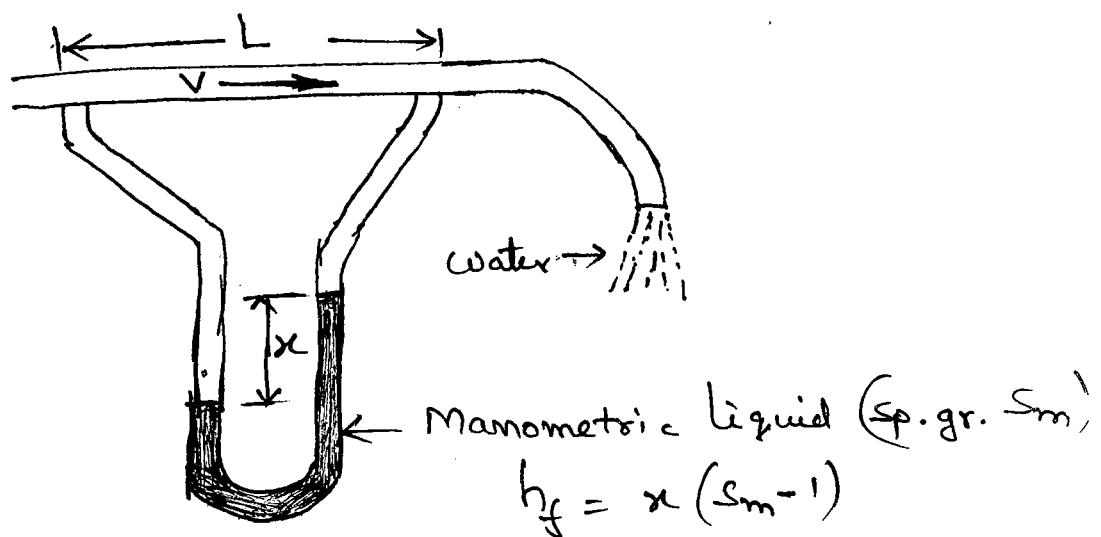


Fig: Apparatus for measuring the Loss of head in a pipe.

- The apparatus required consists of a uniform horizontal pipe of a known diameter, to which a manometer is connected for measuring the loss of head h_f , occurring in a length L , of the pipe.
- The head loss can be obtained from the manometer reading for a particular discharge and the mean velocity v of flow through pipe can be determined from the measured discharge.
- Several values of the head loss can thus be obtained for the corresponding values of the velocities of flow of fluid.

(4)

→ Now if a logarithmic plot of (h_f/L) as ordinate and the velocity V as abscissa is prepared, it will be as shown in fig.

→ From this it will be found that for small values of V , the plot is a straight line with its slope equal to unity.

→ This continues upto certain value of V , represented by point B on the figure, which thus indicates that as long as the velocity is less than the value corresponding to point B,

the head loss due to friction will be directly proportional to the velocity of flow of fluid (i.e., $\frac{h_f}{L} \sim V$)

→ Beyond the point B with increasing velocity, it will be found that there exists certain transition region extending upto point C.

→ During which there is an abrupt increase in the rate at which the loss of head varies.

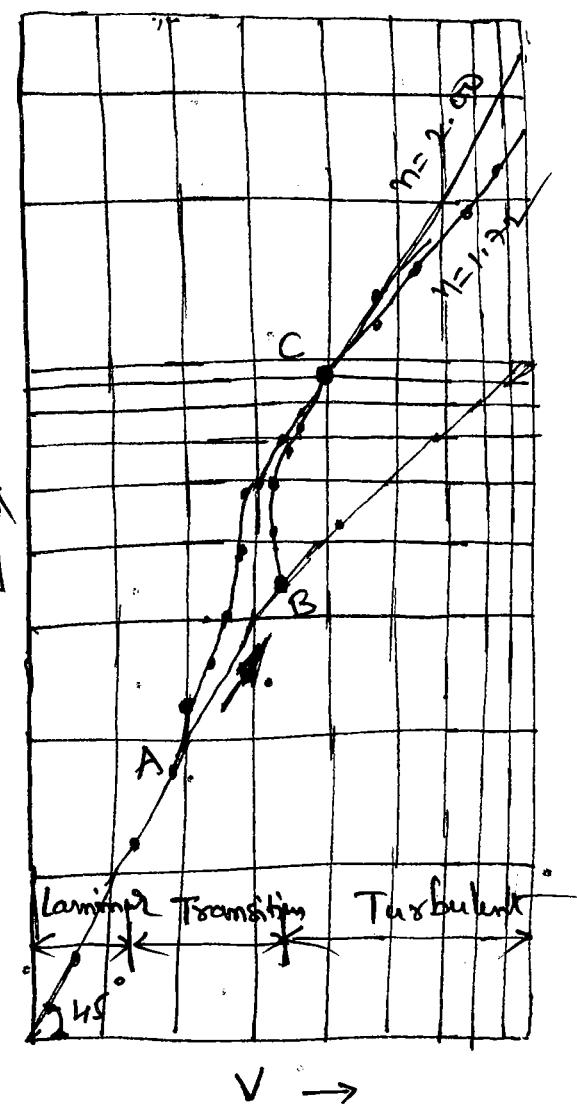


Fig. Plot of $(\frac{h_f}{L})$ vs V showing upper & lower critical points & velocities

- After the region of transition has passed, again the curve obtained is in the form of straight lines with slopes ranging from 1.72 to 2.0.
- However, if the velocity is gradually reduced from a higher value, the line BC will not be retraced.
- Instead the points may be along curve CA, as indicated by arrows in fig.
- The point B is known as the upper critical point and the point A is known as the lower critical point.
- The corresponding velocities are known as upper critical velocity & lower critical velocity.
- It is thus seen that upto point A, the drop in pressure head due to frictional resistance is directly proportional to the mean velocity of flow V , which is the range of laminar flow.
- Beyond point C, the drop in pressure head due to frictional resistance varies as V^n , where n ranges from 1.72 to 2.0, which is the one of turbulent flow.
- Between the points A and C (i.e., in between the regimes of laminar and turbulent flows) lies the transition region as shown in fig.

(5).

Conclusions from figure:

- The upper critical Reynolds number corresponding to point B (i.e., upper limit of laminar flow in pipes) was found by Reynolds to lie between 12000 to 14000.
- But the upper critical Reynolds number is indefinite, being dependent upon initial disturbance affecting the flow, shape of entry of pipe, roughness of pipe wall etc.
- Thus the practical value of upper critical Reynolds number may be considered to lie between 2700 to 4000.
- The lower critical Reynolds number for flow of fluid in pipes corresponding to point A is of greater engineering importance.
- As it indicates a condition below which all turbulence entering the flow from any source will be damped out by viscosity and thus sets a limit below which laminar flow will always occur.
- Experimentally the value of the lower critical Reynolds number has been found to be approximately 2000.
- Between Reynolds numbers 2000 and 4000 the transition region exists.
- The concept of Critical Reynolds number which distinguishes the regimes of laminar and turbulent flow is indeed quite useful in the study of various fluid flow phenomena.

Applying this concept to the flow of water through circular pipes, we may predict that the flow will be laminar if Reynolds number is less than 2000 and turbulent if it is greater than 4000.

It must however be pointed out that critical Reynolds number is very much a function of boundary geometry.

Characteristics of laminar and transitional flow

A flow is said to be laminar when a fluid motion is said to be regular. The various fluid particles move in such a manner that they follow parallel layers (or laminae) with one layer being an entirely bunched up of disordered fluid sliding smoothly over another in a orderly manner, that results in a very and continuous mixing of the fluid. leading to momentum transfer as flow occurs.

It occurs at low velocities but occurs at high velocities.

Here there are no eddies or vortices. In such a flow eddies or vortices of different sizes & shapes are present which move over large distances.

When Reynolds number is less than 1000, Reynolds number is greater than 4000, the flow is turbulent.



Flow through Parallel Plates — Both Plates at Rest

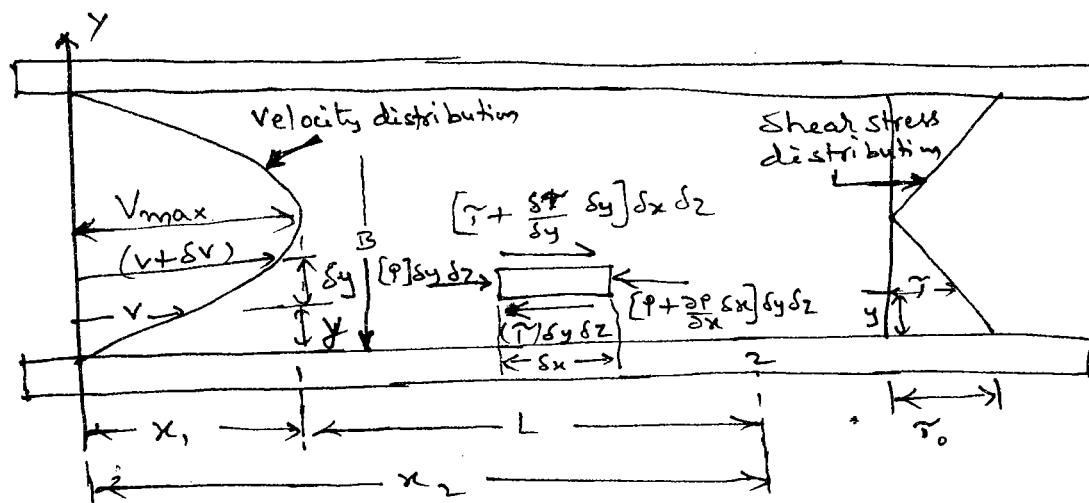


Fig: Laminar flow between two fixed parallel flat plates.

- Consider laminar flow of fluid between two fixed parallel flat plates located at a distance B apart, as shown in figure.
- A small rectangular element of the fluid of length δx is as free-body as shown in figure.
- Let the lower face of the element be at a distance y from the lower plate and here let the velocity be v .
- At the upper face of the element which is at a distance of $(y + \delta y)$ from the lower plate, Let the velocity be $(v + \delta v)$
- If δv is positive, the faster moving fluid just above the upper face of the element exerts a forward force across the upper face.
- Similarly, the slower moving fluid adjacent to the lower face tends to retard its motion i.e., it exerts a backward force on the lower face.
- Thus there are shear stresses of magnitude τ on the lower face and $(\tau + \frac{\partial \tau}{\partial y} \delta y)$ on the upper face of the element in the directions as shown in figure.

- In order to balance the shearing forces in the fluid a pressure gradient in the direction of flow must be maintained.
- Thus if P is the pressure intensity at the left face of the element, $(P + \frac{\partial P}{\partial x} \delta x)$ then will be the pressure intensity on the right face of the element.
- If the width of the element in the direction perpendicular to the paper is δz , the total force acting on the element towards the right is
- $$[P - (P + \frac{\partial P}{\partial x} \delta x)] \delta y \delta z + \left[\left(\tau + \frac{\partial \tau}{\partial y} \delta y \right) - \tau \right] \delta x \delta z.$$
- But for steady and uniform flow, there is no acceleration and hence this total force must be equal to zero.
- $$-\frac{\partial P}{\partial x} \cdot \delta x \delta y \delta z + \frac{\partial \tau}{\partial y} \delta x \delta y \delta z = 0$$
- Dividing by the volume of the element ($\delta x \delta y \delta z$), we get $\frac{\partial P}{\partial x} = \frac{\partial \tau}{\partial y}$.
- Again according to Newton's Law of viscosity for laminar flow the shear stress $\tau = \mu \cdot \frac{du}{dy}$.
- Hence by substitution in the above equation the following differential equation for laminar flow is obtained,
- $$\frac{\partial P}{\partial x} = \mu \cdot \frac{\partial^2 v}{\partial y^2}.$$
- Since, $(\frac{\partial P}{\partial x})$ is independent of y , integrating the above equation twice with respect to y gives
- $$v = \frac{1}{\mu} \left(\frac{\partial P}{\partial x} \right) \frac{y^2}{2} + C_1 y + C_2 \rightarrow \textcircled{a}$$

⑦ → The two constants of integration c_1 and c_2 may be evaluated by means of two boundary conditions.

→ There being no slip of fluid at the two solid boundary surfaces of the plates,

$$v = 0 \quad \text{at } y = 0$$

$$\text{and } c_2 = 0$$

$$v = 0 \quad \text{at } y = B$$

→ Now we get the value of first constant of integration as $c_1 = -\frac{B}{2\mu} \left(\frac{\partial P}{\partial x} \right)$

→ Introducing these values in Equation (a) the following equation for the velocity distribution is obtained

$$v = \frac{1}{2\mu} \left(-\frac{\partial P}{\partial x} \right) (By - y^2) \rightarrow (b)$$

→ Equation (b) indicates that the velocity distribution curve for the laminar flow between parallel flat plate is a parabola with its vertex being mid-way between the plates.

→ The negative sign for the pressure gradient indicates that there is a drop in pressure in the direction of flow.

→ The maximum velocity v_{max} which occurs at the mid point between the plates,

$$v_{max} = \frac{B^2}{8\mu} \left(-\frac{\partial P}{\partial x} \right) \quad (8), \quad v = \frac{2}{3} v_{max}$$

→ Pressure gradient is given by, $\left(-\frac{\partial P}{\partial x} \right) = \frac{12\mu v}{B^2}$

→ Drop in pressure head (h_f) is given as,

$$h_f = \frac{P_1 - P_2}{w} = \frac{12 \mu V L}{w B^2}$$

→ The distribution of shear stress in the flowing fluid across any section of the parallel plates may be determined by substituting Equation (b) into Newton's Law of Viscosity.

$$\text{Thus, } \tau = \mu \frac{\partial v}{\partial y}$$

$$\therefore \tau = \mu \frac{\partial}{\partial y} \left[\frac{1}{2\mu} \left(-\frac{\partial p}{\partial x} \right) (By - y^2) \right]$$

$$\therefore \tau = \left(-\frac{\partial p}{\partial x} \right) \left(\frac{B}{2} - y \right)$$

* Flow through Long pipes:

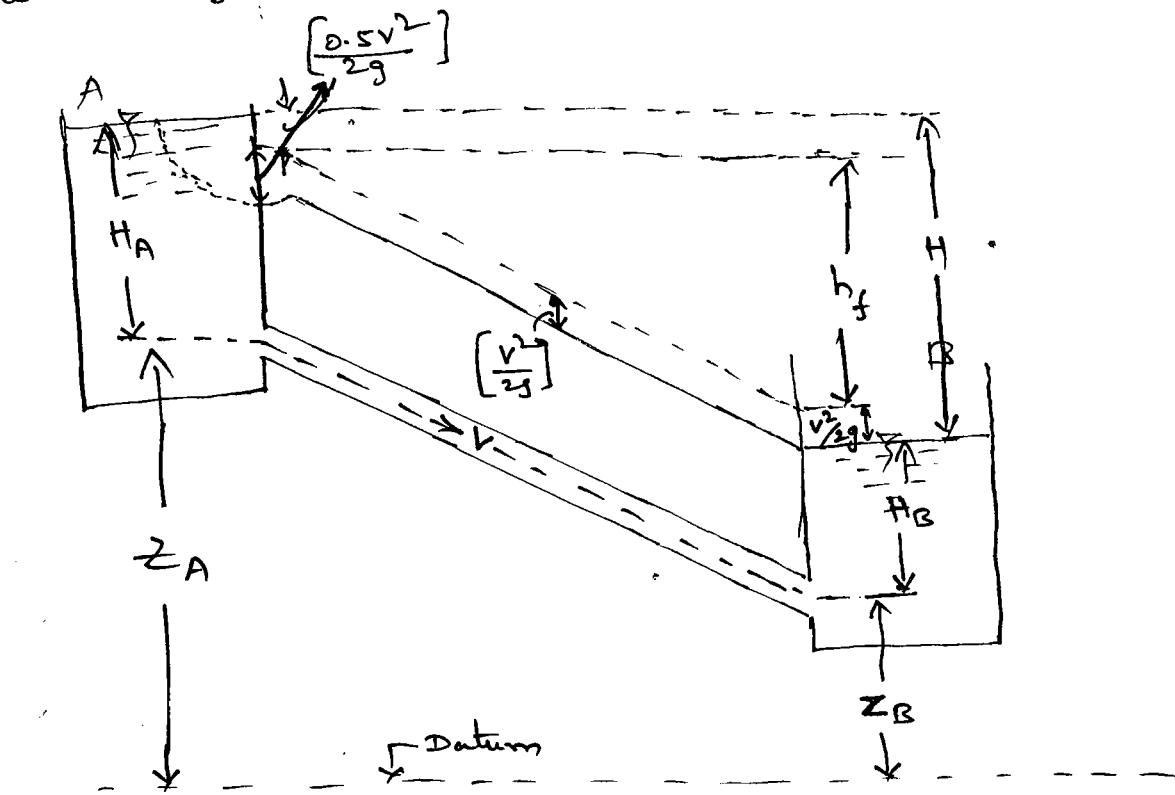


Fig: Flow through a long pipe

8) → consider a long pipeline of diameter D and length L carrying liquid from a reservoir A to another reservoir B, as shown in figure.

→ Let H_A and H_B be the constant heights of the liquid surfaces in the reservoirs A and B respectively above the centre of the pipe

→ Further let Z_A and Z_B be the heights of the centres of the pipe ends connected to the reservoirs A and B respectively.

→ Now if V is the mean velocity of flow through the pipe then the head loss due to friction (h_f),

$$h_f = \frac{f L V^2}{2gD}$$

where f - friction factor

L - Length of Pipe (m).

V - Velocity of fluid flow (m/s),

g - Acceleration due to gravity (m/s²),

D - Diameter of pipe (in metres)

→ Head loss at the entrance of pipe, = $0.5 \frac{V^2}{2g}$

→ Head loss at the exit of pipe, = $\frac{V^2}{2g}$

→ Applying Bernoulli's equation between points ① and ② in the reservoirs A and B respectively, we obtain

$$H_A + Z_A = H_B + Z_B + 0.5 \frac{V^2}{2g} + \frac{f L V^2}{2gD} + \frac{V^2}{2g}$$

$$\text{Q) } (H_A + Z_A) - (H_B + Z_B) = \frac{V^2}{2g} \left(1.5 + \frac{fL}{D} \right)$$

$$\text{But } (H_A + z_A) - (H_B + z_B) = H$$

where H is the difference in the liquid surfaces in the reservoirs A and B. Thus

$$H = \frac{v^2}{2g} \left(1.5 + \frac{fL}{D} \right) \rightarrow ①$$

→ Equation ① indicates that the difference in the liquid surfaces in the two reservoirs at the two ends of the pipe is equal to the sum of the various head losses.

→ From this equation the unknown velocity may be computed

→ If the pipe is long (say, more than 1000 times the diameter), the loss of head due to friction will be very large as compared with the minor losses which may then be neglected, thereby simplifying the expression as,

$$H = \frac{fL v^2}{2g D}; \quad \therefore v = \sqrt{\frac{2g HD}{fL}}$$

→ If the pipe in figure instead of discharging into the reservoir B, discharges into the atmosphere the equation would then be,

$$(H_A + z_A) = z_B + 0.5 \frac{v^2}{2g} + \frac{fL v^2}{2g D} + \frac{v^2}{2g}$$

$$\therefore H = \frac{v^2}{2g} \left(1.5 + \frac{fL}{D} \right)$$

where H is the height of the liquid surface in the reservoir A above the outlet end of the pipe.

Q. Flow through Inclined Pipes

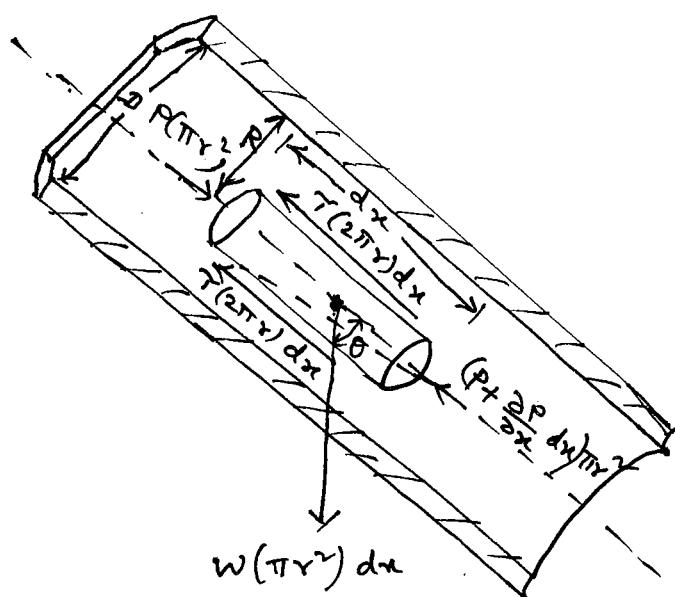


Fig: Laminar flow through an inclined pipe.

- When the pipe having laminar flow of fluid is inclined as shown in figure in addition to pressure and viscous force gravity forces will also become effective.
- Consider a small concentric cylindrical fluid element, its specific weight of the fluid is w , the weight of the element is $[w(\pi r^2)dx]$ acting vertically downward.
- Its component in the direction of flow becomes $[w(\pi r^2)dx] \cos\theta$ in which x -direction is taken parallel to the axis of pipe.
- Now if z -direction is taken vertically upward, let dz represents a change in elevation corresponding to length dx of the fluid element.

→ Thus from figure, we get, $\cos\theta = -\frac{dz}{dx} = -\frac{\partial z}{\partial x}$

→ In which the minus sign has been considered because the elevation z decreases as the distance x increases.

→ The component of the weight of the fluid element may thus be written as,

$$[\omega(\pi r^2)dx] \cos\theta = [\omega(\pi r^2)dx] \left(-\frac{dz}{dx}\right) = -\omega(\pi r^2) dz \rightarrow (1)$$

→ The other forces acting on the fluid element are the shear and pressure forces as shown in figure.

→ In the absence of any acceleration, sum of all the forces acting on the element in the direction of flow must be equal to zero. Thus

$$P\pi r^2 - \left(P + \frac{\partial P}{\partial x} dx\right)\pi r^2 - \omega\pi r^2 dz - \tau(2\pi r)dx = 0$$

$$\tau = -\frac{\partial}{\partial x} \left(P + \omega z\right) \frac{r}{2} = -\frac{\partial}{\partial x} \left(\frac{P}{\omega} + z\right) \frac{r}{2} \rightarrow (2)$$

$$\tau = -\omega \frac{\partial h}{\partial x} \frac{r}{2} \rightarrow (3)$$

→ In which, $h = \left[\frac{P}{\omega} + z\right]$ represents piezometric head.

→ For pipes, according to Newton's law of viscosity

$$\tau = -\mu \frac{\partial v}{\partial r}$$

→ We now have,

$$-\mu \frac{\partial v}{\partial r} = -\omega \frac{\partial h}{\partial x} \frac{r}{2}$$

$$(d) \quad \frac{\partial v}{\partial r} = \frac{\omega}{\mu} \frac{\partial h}{\partial x} \frac{r}{2}$$

* The velocity profile for flow in an inclined pipe is given as,

$$v = \frac{1}{4\mu} \times w \left(-\frac{\partial h}{\partial x} \right) \left(R^2 - r^2 \right)$$

Problems

1. A pipeline 0.225m in diameter and 1580m long has a slope of 1 in 200 for the first 790m and 1 in 100 for the next 790m. The pressure at the upper end of the pipeline is 107.91 kpa and at the lower end is 53.955 kpa. Taking $f=0.032$ determine the discharge through the pipe.

E.P.: Given Data:

Diameter of pipe (d) = 0.225m

Length of pipe (L) = 1580m.

Pressure at upper end (P_1) = 107.91 kpa

Pressure at lower end (P_2) = 53.955 kpa

Friction factor (f) = 0.032

Data to be calculated:

Discharge through pipe (Q)

Solution: Assuming the datum to be passing through the lower end of the pipe, the datum head for the upper end of the pipeline is

$$Z_1 = \frac{790}{200} + \frac{790}{100} = (3.95 + 7.90) = 11.85 \text{ m.}$$

Applying Bernoulli's equation between the upper and the lower ends of the pipeline,

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$\text{But } V_1 = V_2 = V$$

Then the loss of head due to friction,

$$h_f = \frac{f L V^2}{2g D} = \frac{0.032 \times 1580 \times V^2}{2 \times 9.81 \times 0.225} = 11.45 V^2$$

Thus by substitution, we get,

$$\frac{1.07.91 \times 10^3}{9810} + \frac{V^2}{2g} + 11.85 = \frac{53.955 \times 10^3}{9810} + \frac{V^2}{2g} + 0 + 11.45 V^2$$

$$\text{or } 11.45 V^2 = 17.35$$

$$\text{or } V = \sqrt{\frac{17.35}{11.45}} = 1.23 \text{ m/s}$$

$$\begin{aligned} \therefore \text{Discharge, } Q &= \frac{\pi D^2}{4} \times V \\ &= \frac{\pi (0.225)^2}{4} \times 1.23 \\ &= 0.0489 \text{ m}^3/\text{s} \end{aligned}$$

Result:

$$\text{Discharge through pipe (Q)} = 0.0489 \text{ m}^3/\text{s.}$$

(11)

3. A 0.3m diameter pipe 2340m long is connected with a reservoir whose surface is 72m above the discharging end of the pipe. If for the last 1170m, a second pipe of the same diameter be laid beside the first and connected to it, what would be the increase in the discharge? Take $f = 0.02$.

Given Data:

$$\text{Diameter of pipe } (d) = 0.3 \text{ m}$$

$$\text{Length of pipe } (L) = 2340 \text{ m}$$

$$\text{Elevation } (z), \text{ head } (H) = 72 \text{ m}$$

$$\text{Friction factor } (f) = 0.02$$

Data to be calculated:

$$\text{Increase in discharge } (Q_2 - Q_1)$$

Solution:

We know that

$$H = \left(1.5 + \frac{fL}{D} \right) \frac{V^2}{2g}$$

Substituting the known values, & solving we get,

$$7.2 = \left(1.5 + \frac{0.02 \times 2340}{0.30} \right) \frac{V^2}{2 \times 9.81}$$

$$V = \sqrt{\frac{7.2 \times 2 \times 9.81}{157.5}} = 3 \text{ m/s}$$

Discharge is given by

$$Q_1 = \frac{\pi D^2}{4} \times V = \frac{\pi (0.30)^2}{4} \times 3 = 0.212 \text{ m}^3/\text{s}$$

In the second case let Q_2 be the total discharge since for the second half of the length there are two parallel pipes of the same diameter, each pipe will carry discharge equal to $\left(\frac{Q_2}{2}\right)$. Also the velocity of flow in each pipe will be equal to half the velocity of flow V in the first half of the length.

Thus applying Bernoulli's equation between the water surface in the reservoir and the outlet of pipe, we get,

$$\gamma_2 = 0.5 \frac{V^2}{2g} + \underbrace{0.02 \times 1170 \times V}_{2g \times 0.30} * \frac{0.02 \times 1170 \times \left(\frac{V}{2}\right)^2}{2g \times 0.30} + \frac{\left(\frac{V}{2}\right)^2}{2g}$$

$$\gamma_2 = \frac{V^2}{2g} [0.5 + 78.0 + 0.25]$$

$$V = V_2 = 3.79 \text{ m/s.}$$

$$\therefore Q_2 = \frac{\pi}{4} (0.30)^2 \times 3.79 = 0.268 \text{ m}^3/\text{s}$$

$$\begin{aligned} \text{Increase in discharge} &= Q_2 - Q_1 \\ &= 0.268 - 0.212 \\ &= 0.056 \text{ m}^3/\text{s} \end{aligned}$$

$$\text{Percentage increase in discharge} = \frac{0.056}{0.212} \times 100 \\ = 26.42\%$$

$$\text{Result: Increase in discharge } (Q_2 - Q_1) = 0.056 \text{ m}^3$$

(12)

- ③ Two parallel plates kept 0.1m apart have laminar flow of oil between them with a max. velocity of 1.5 m/s. Calculate the discharge per metre width, the shear stress at the plates, the difference in pressure in Pascal's between two points 20 m apart, the velocity gradient at the plates and velocity at 0.02 m from the plate take viscosity of oil to be 2.453 Ns/m^2 .

Given Data:

$$\text{Distance between parallel plates (d)} = 0.1\text{m}$$

$$\text{Maximum velocity } (V_{\max}) = 1.5 \text{ m/s}$$

$$\text{Viscosity of oil } (\mu) = 2.453 \text{ Ns/m}^2$$

Data to be calculated:

$$\text{Discharge per metre width } (q)$$

$$\text{Shear stress at plates } (\tau_0)$$

$$\text{Difference in pressure } (P_1 - P_2)$$

$$\text{Velocity } (v) \text{ at a distance } 0.02\text{m from plate.}$$

Solution:

In this case the mean velocity of flow V is equal to two-thirds of the max. velocity.

$$V = \frac{2}{3} (1.5) = 1.0 \text{ m/s}$$

The discharge q per metre width plate is given by

$$q = VB = (1.0 \times 0.1) = 0.1 \text{ m}^3/\text{s} \text{ per m.}$$

$$\text{We know that, } V = \frac{12 \mu V}{B^2} = \frac{12 \times 2.453 \times 1.0}{(0.1)^2} = 2943.6 \frac{\text{m}}{\text{s}^2} \text{ m/s.}$$

The shear stress at the plates is given by

$$\tau_0 = \left(-\frac{\partial p}{\partial x} \right) \frac{B}{2} = \frac{2943.6 \times 0.1}{2} = 147.18 \frac{N}{m^2}$$

The pressure difference between the two points is given by

$$(P_1 - P_2) = \frac{12 \mu V}{B} = \frac{12 \times 2.453 \times 1.0 \times 2.0}{(0.1)^2}$$

$$= 58872 \frac{N}{m^2} = 58.872 \frac{kN}{m^2}$$

The shear stress at plates is also given by

$$\tau_0 = \mu \left(\frac{\partial v}{\partial y} \right)_{y=0}$$

As such the velocity gradient at the plates is given by

$$\left(\frac{\partial v}{\partial y} \right)_{y=0} = \frac{\tau_0}{\mu} = \frac{147.18}{2.453} = 60 \frac{1}{s}.$$

The velocity v at a distance of $0.02 m$ from the plate is given by

$$v = \frac{1}{2\mu} \left(-\frac{\partial p}{\partial x} \right) (By - y^2)$$

$$= \frac{1}{2 \times 2.453} (2943.6) \left[(0.1 \times 0.02) - 0.02^2 \right]$$

$$= 0.96 \text{ m/s}$$

Result:

Discharge per metre width (Q) = $0.3 \text{ m}^3/\text{s}$ per turn

Shear stress at plates (τ_0) = 147.18 N/m^2

Difference in pressure ($P_1 - P_2$) = 58.872 kN/m^2

Velocity (V) at a distance $0.02 m$ from plate = 0.96 m/s

Lecture HandoutClosed Conduit FlowLearning Objectives.

At the end of this topic, you will be able to:

- Understand Laws of fluid friction
- Derive Darcy - Weisbach Equation
- Explain losses in pipe flow
- Describe about pipes arranged in series & parallel
- Draw the total energy line and hydraulic gradient line
- Explain Moody's chart
- solve problems based on formulae and concepts.

Learning outcomes:

By the end of this topic, you will be able to know the laws of fluid friction

Evaluate Darcy - weisbach equation

Understand the losses in pipe flow

know about pipes arranged in series and parallel

Illustrate total energy line and hydraulic gradient line

Understand Moody's chart

Evaluate problems based on formulae and concepts.

→ Laws of Fluid friction:-

- The frictional resistance offered to the flow depends on the type of flow..
- Different laws are obeyed by the frictional resistance in the laminar and turbulent flows.

- On the basis of the experimental observations the laws of fluid friction for the two types of flows are
- Laws of fluid friction for laminar flow
 - Laws of fluid friction for Turbulent Flow.
- The frictional resistance in laminar flow is as follows

Proportional to Velocity of flow

↓
Independent of Pressure

↓
Proportional to area of surface in contact

↓
Independent of nature of surface in contact

↓
Greatly affected by variation of temperature on flowing fluid

⇒ Laws of fluid friction for Turbulent flow

→ The frictional resistance in case of turbulent flow is as follows.

Proportional to $(\text{velocity})^n$, where n varies from 1.72 to 2.7

↓
Independent of pressure

↓
Proportional to density of flowing fluid & area of surface in contact

↓
slightly affected by variation of temperature of flowing fluid

↓
Dependent on nature of surface in contact

Froude's Experiments

→ W. Froude conducted a series of experiments to investigate frictional resistance offered to the flowing water by different surfaces.

Experimental setup:

- The experiments were conducted in a tank about 100 m (300 ft) long, 11 m (36 ft) broad and 3 m (10 ft) deep containing water.
- Thin wooden boards about 5 mm ($\frac{3}{16}$ in) thick; 0.475 m (19 in) wide and lengths varying from 0.6 m (2 ft) to 1.5 m (5 ft) were towed end wise in this tank by connecting them to a carriage running on rails provided on the sides of the tank.
- The carriage was hauled along at speeds varying from 30 m (100 ft) to 300 m (1000 ft) per minute, by means of a wire rope passing around a drum.
- The boards were towed in a completely submerged position such that the upper edge was about 0.45 m below the water surface in the tank and the force required to tow the board being measured.
- In order to develop the surfaces of different types the surfaces of the boards were covered with varnish, tinfoil, calico and sand in turn.

Conclusions:

- ⇒ From the results of these experiments Froude derived the following.
1. The frictional resistance varies approximately with the square of the body.
 2. The frictional resistance varies with the nature of surface.
 3. The frictional resistance per unit area of the surface decreases as the length of the board increases but is constant for long lengths.
- Thus if f' is the frictional resistance per unit area of given surface at unit velocity, A is the area of wetted surface and V is the velocity, then the total friction resistance F is given by
- $$F = f' A V^n$$
- Assuming the index, $n=2$.
- $$F = f' A V^2$$
- ⇒ Equation for Head Loss in Pipes Due to Friction - Darcy - Weisbach Equation.
- Consider a horizontal pipe of cross-sectional area A carrying a fluid with a mean velocity V .
- Let 1 and 2 be the two sections of the pipe L distance apart, where the intensities of pressure be P_1 and P_2 respectively.

→ By applying Bernoulli's equation between the sections 1 and 2, we obtain

$$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + z_2 + h_f$$

Since $V_1 = V_2 = V$ and $z_1 = z_2$

$$\text{Loss of head } = h_f = \frac{P_1}{\omega} - \frac{P_2}{\omega}$$

→ i.e., the pressure intensity will be reduced by the frictional resistance in the direction of flow.

→ The difference of pressure heads between any two sections is equal to the loss of head due to friction between these sections.

→ Let f' be the frictional resistance per unit area at unit velocity, then frictional resistance

$$= f' \times \text{Area} \times V^n$$

$$= f' \times PL \times V^n$$

→ Where P is the wetted perimeter of the pipe.

→ The pressure forces at the sections 1 and 2 are $(P_1 A)$ and $(P_2 A)$, respectively. Thus resolving all the forces horizontally, we have

$$P_1 A = P_2 A + \text{frictional resistance}$$

$$\text{or } (P_1 - P_2)A = f' \times PL \times V^n$$

$$\text{or } (P_1 - P_2) = f' \times \frac{P}{A} \times LV^n$$

→ Dividing both sides by the specific weight ω of the flowing fluid.

$$\frac{(P_1 - P_2)}{\omega} = \frac{f'}{\omega} \times \frac{P}{A} L V^2$$

but $h_f = \frac{(P_1 - P_2)}{\omega}$, then

$$h_f = \frac{f'}{\omega} \times \frac{P}{A} L V^2$$

→ The ratio of the cross sectional area of the flow (wetted area) to the perimeter in contact with the fluid (wetted perimeter) i.e. $\left(\frac{A}{P}\right)$ is called Hydraulic Mean Depth (H.M.D) or hydraulic radius and is represented by m or R

$$\rightarrow \text{Then, } h_f = \frac{f'}{\omega} \times \frac{L V^2}{m}$$

$$\rightarrow \text{For pipes running full, } m = \frac{A}{P} = \frac{\left(\frac{\pi D^2}{4}\right)}{\pi D} = \frac{D}{4}$$

\rightarrow Substituting this in the equation for h_f and assuming $n=2$

$$\therefore \text{we have, } h_f = \frac{4f'}{\omega} \cdot \frac{L V^2}{D}$$

$$\text{putting } \frac{4f'}{\omega} = \frac{f}{2g}$$

$$h_f = \frac{f L V^2}{2g D} \quad \dots \quad (1)$$

\rightarrow where f is known as friction factor, which is dimensionless quantity.

\rightarrow Equation 1 is known as Darcy-Weisbach equation which is commonly used for computing the loss of head due to friction in pipes.

\rightarrow It may be noted that the head loss due to friction is also expressed in terms of the velocity head $\left(\frac{V^2}{2g}\right)$ corresponding to the mean velocity.