

## UNIT-II

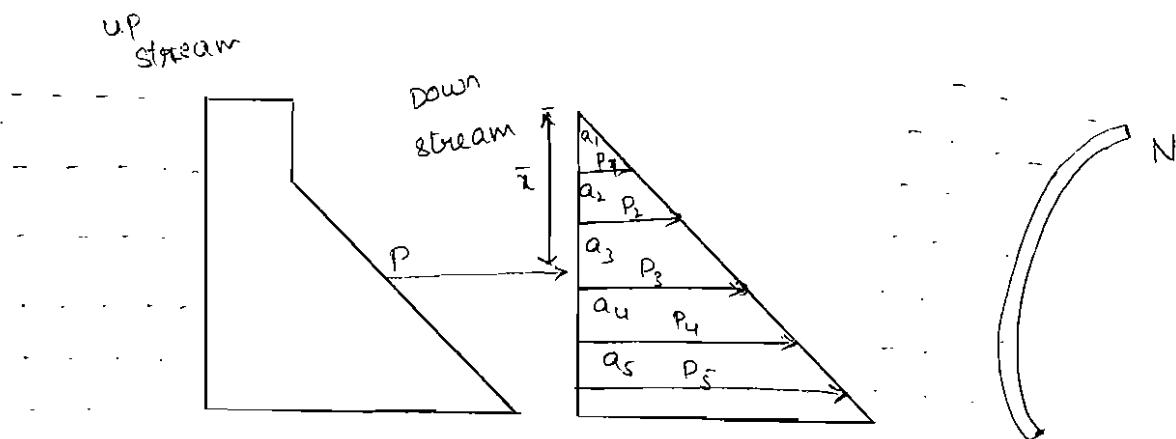
Fluid statics

It consists of a thin metallic tube having deep circumferential curvations called "Bellows". As the pressure changes these bellows expands/ contracts and cause movements of pointer on a graduated circular disk as shown in figure no

\* Hydrostatic forces on surfaces:-

$$\text{Total pressure} = P = p_1 a_1 + p_2 a_2 + p_3 a_3 + \dots + p_n a_n$$

$\bar{x}$  = centre of pressure.



Hydrostatic force

The pressure exerted by the static fluid on the surfaces with which it comes into contact is known as Hydrostatic pressure.

when fluid is at rest condition velocity gradient and shear stress ( $\tau$ ) is equal to zero.

### Total pressure:

It is the sum of the intensities of pressure on different strips of surface entered by the static fluid.

$$P = P_1 a_1 + P_2 a_2 + P_3 a_3 + \dots + P_n a_n$$

where,  $P$  is known as total pressure.

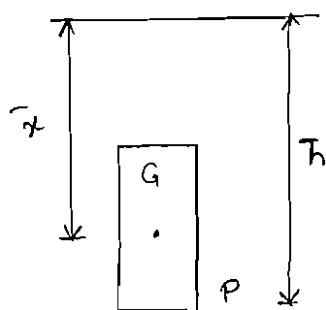
$P_i a_i$  is the intensities of pressure.

### centre of pressure:

It is defined as the point through which the total pressure is assumed to act is known as centre of pressure.

(d)

The point of application of total pressure is known as centre of pressure. It is denoted by  $\tilde{h}$ .



## \* Types of immersed bodies:

There are mainly 4 types of immersed bodies

They are i, Horizontally immersed plane surface.

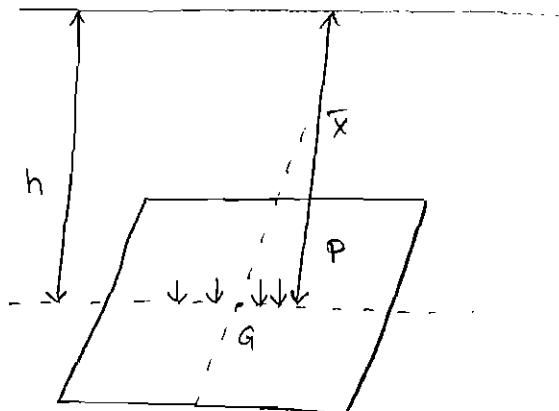
ii, Vertically immersed plane surface.

iii, Inclined immersed plane surface.

iv, Curved surfaces →(d) Submerged

i, Horizontally immersed plane Surfaces :-

free Surface of liquid.



consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid the pressure intensity will be equal on the entire surface and is equal to

$$p = \rho gh$$

$$\phi = w \cdot h$$

total pressure is

$$\underline{P} = p_1 a_1 + p_2 a_2 + \dots$$

$$\underline{P} = w \cdot h \times A$$

$$\underline{P} = w \cdot A \times \bar{x}$$

Centre of pressure:

$$h = \bar{x}$$

Since, every point on the surface of the body is at equal distance from the free surface of the liquid. Hence, the intensity of pressure on the surface is constant.

Hence  $\bar{x} = h$  ( $\because h = \bar{h}$ )

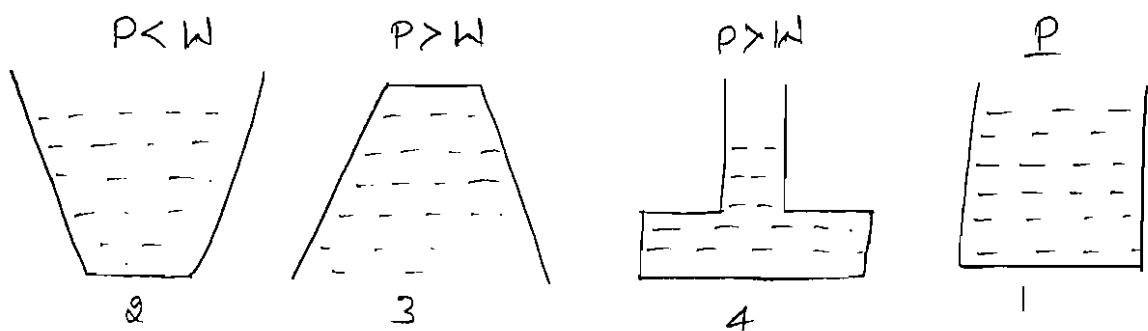
(The figure shows a tank.

a, Total pressure on the bottom of the tank.

b, wt. of water in the tank.)

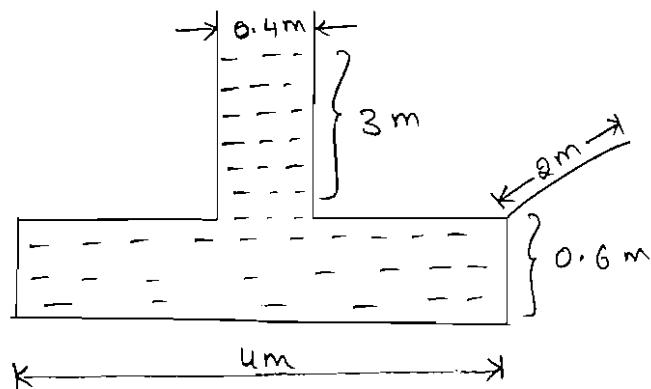
\* The figure shows a tank.

- Total pressure on the bottom of the tank
- wt. of water in the tank.
- hydrostatic paradox b/n results ① & ② if width of the tank is 8m.



### Hydrostatic paradox:

The apparent contradiction in the hydrostatic force on the container base and weight of the liquid inside the container is known as hydrostatic paradox.



$$a, P = (\rho \cdot g \cdot h) \times A$$

$$= 1000 \times 9.81 \times (3 + 0.6) (ux2)$$

$$= 282.528 N$$

$$= 282.52 KN$$

b, Weight of the fluid contained in the container

$$= \omega \times V$$

$$= (1000 \times 9.81) \times [3 \times 0.4 \times 2 + ux2 \times 0.6]$$

$$= 70.632 N$$

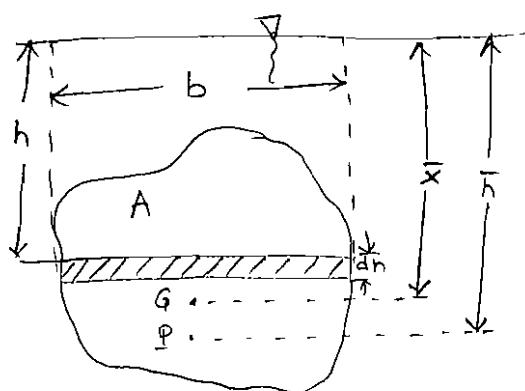
$$= 70.632 KN$$

c, Hydrostatic paradox

$$= (282.52 - 70.632) KN$$

$$= 211.88 KN$$

\* vertical plane surface submerged in liquid;



consider a plane vertical surface immersed in a liquid as shown in figure.

Let

$A$  = total area of the surface.

$\bar{h}$  = distance of centre of pressure from the free surface of the liquid.

$\bar{x}$  = distance of centroid of the area from the free surface of the liquid

$G$  = (first) centroid of plane surface.

$P$  = centre of pressure.

$B$  = width of the elementary strip

$dh$  = depth of the elementary strip

$h$  = centroid of the elementary strip

consider a strip of thickness  $dh$  and width  $b$

at a depth  $h$  from the free surface of the liquid.

$$P = p_1 a_1 + p_2 a_2 + p_3 a_3 + \dots + p_n a_n$$

$$P = w \cdot h$$

$$P = p \cdot g \cdot h \quad [dA = b \cdot dh \text{ & } \int dA \cdot h = \bar{x}]$$

↓  
strip area.

$$dF = (p \cdot g \cdot h) \cdot dA$$

$p \quad A$

$$\int dF = \int \rho g \cdot h \cdot b \cdot dh$$

$$F = \rho g \cdot \int h \cdot b \cdot dh$$

$$F = \rho g \cdot \int h \cdot dA$$

$$F = \rho g \cdot \int dA \cdot h$$

$F = \rho g \times \int \text{Area of Surface} \times \text{distance of centroid of elementary strip}$

$$\int dA = A$$

$$\int h = \bar{x}$$

∴ Total pressure face acting on the liquid is

$$F = \rho g \cdot A \cdot \bar{x}$$

Total pressure

$$F = \omega \cdot A \cdot \bar{x}$$

\* centre of pressure:

centre of pressure is obtained by using varignon's theorem.

Moment of resultant force = algebraic sum of moment of its components.

$$F \times \bar{x} = df \times h$$

$$dF = \rho g \cdot h \cdot dA$$

$$dF = \rho \cdot g \cdot h \cdot b \cdot dh$$

$$Fx\bar{h} = \int (\rho \cdot g \cdot h \cdot b \cdot dh) h$$

$$Fx\bar{h} = \int \rho \cdot g \cdot h^2 \cdot b \cdot dh$$

$$Fx\bar{h} = \rho g \int dA \cdot h$$

$$\therefore Fx\bar{h} = \rho \cdot g \cdot I_0$$

$$I_0 = I_G + A \cdot \bar{x}^2$$

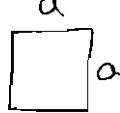
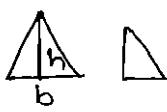
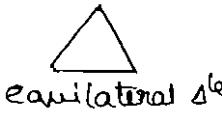
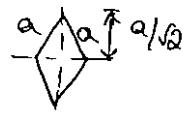
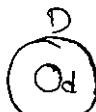
$$Fx\bar{h} = \rho \cdot g \cdot (I_G + A \cdot \bar{x}^2)$$

$$\rho \cdot g \cdot A \bar{x} \bar{h} = \rho \cdot g \cdot (I_G + A \cdot \bar{x}^2)$$

$$\therefore \bar{h} = \frac{\rho \cdot g \cdot (I_G + A \cdot \bar{x}^2)}{\rho g \cdot A \cdot \bar{x}}$$

$$\bar{h} = \frac{I_G + A \bar{x}^2}{A \cdot \bar{x}}$$

$$\boxed{\therefore \bar{h} = \frac{I_G}{A \cdot \bar{x}} + \bar{x}}$$

| Sl. No. | Shape   | Area                       | Centroid from top                                    | $I_{xx}$                    |
|---------|---|----------------------------|--|-----------------------------|
| 1.      |    | $bd$                       | $d/2$  | $\frac{bd^3}{12}$           |
| 2.      |    | $a^2$                      | $a/2$  | $a^4/12$                    |
| 3.      |    | $\frac{1}{2}bh$            | $\frac{2h}{3}$                                       | $\frac{bh^3}{36}$           |
| 4.      |    | $\frac{1}{2}bh$            | $h/3$  | $\frac{bh^3}{36}$           |
| 5.      |  | $\frac{\pi D^2}{4}$        | $\frac{D}{2}$  | $\frac{\pi D^4}{64}$        |
| 6.      |  | $\frac{\sqrt{3}}{4}a^2$    | $\frac{2h}{3}$<br>(where $h = \frac{\sqrt{3}}{2}a$ ) | $\frac{bh^3}{36}$           |
| 7.      |  | $a^2$                      | $\frac{a}{\sqrt{2}}$                                 | $\frac{a^4}{12}$            |
| 8.      |  | $\frac{\pi(D^2 - d^2)}{4}$ | $\frac{D}{2}$  | $\frac{\pi(D^4 - d^4)}{64}$ |

Fd    vertically    immersed    bodies

$$\begin{aligned}
 P &= \omega \cdot A \cdot \bar{x} \\
 \bar{h} &= \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}}
 \end{aligned}$$

Problems:

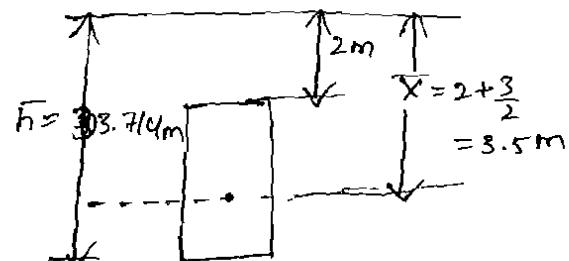
- 1) calculate the total pressure and center of pressure on a velocity immersed rectangular body  $2m \times 3m$  immersed such that the top is at a depth of 2m from the free surface of a liquid specific wt of water =  $10 \text{ kN/m}^3$

sol

$$\text{Area} = 2 \times 3 = 6 \text{ m}^2$$

$$\bar{x} = 2 + \frac{3}{2} = 3.5 \text{ m}$$

$$I_{xx} = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = \frac{8 \times 3}{8} \\ = 4.5 \text{ m}^4$$



$$P = w \cdot A \bar{x}$$

$$= 10 \times 6 \times 3.5$$

$$= 210 \text{ kN}$$

$$h = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}}$$

$$= 3.5 + \frac{4.5}{6(3.5)}$$

$\therefore h = 3.714 \text{ m}$  from the free surface of the liquid

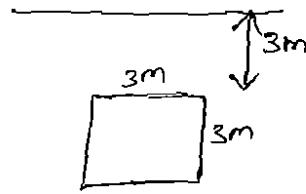
- 2) A square body of size  $3m \times 3m$  is immersed vertically such that the top is at a depth of  $3m$  from the free surface of liquid specific wt of water =  $10 \text{ kN/m}^3$

Sol

$$\text{Area} = 9 \text{ m}^2$$

$$\bar{x} = 3 + \frac{3}{2} = 4.5 \text{ m}$$

$$I_{xx} = \frac{24}{12} = 6.75 \text{ m}^4$$



$$P = \rho \cdot g \cdot \bar{x} = 10 \times 9 \times 4.5 = 405 \text{ kN}$$

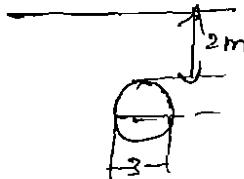
$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} = 4.5 + \frac{6.75}{9 \times 4.5} = 4.66 \text{ m}$$

= 4.66 m from the free surface of the liquid

- 3) A circular body 3m in diameter is immersed vertically such that the top is at a depth of 2m from the free surface of the liquid calculate the total pressure & center of pressure.

Sol

$$\begin{aligned} \text{Area} &= \frac{\pi D^2}{4} = \frac{\pi \times 3^2}{4} \\ &= 7.06 \text{ m}^2 \end{aligned}$$



$$\bar{x} = 2 + \frac{3}{2} = 3.5 \text{ m}$$

$$I_{xx} = \frac{\pi D^4}{64} = 3.97 \text{ m}^4$$

$$\begin{aligned} P &= 10 \times 7.06 \times 3.5 \\ &= 247.38 \text{ kN} \end{aligned}$$

$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} = 3.5 + \frac{3.97}{7.06 \times 3.5}$$

= 3.66 m from the free surface of the liquid

H) A triangle body of base width 3m and height 2.4m is immersed vertically such that the vertex is at a depth of 1.6m from the free surface of a liquid specific wt of water = 10 kN/m<sup>3</sup>

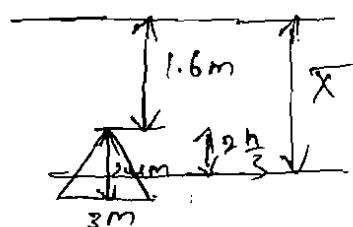
Sol

$$\text{Area} = \frac{1}{2} \times 3 \times 2.4 \\ = 3.6 \text{ m}^2$$

$$\bar{x} = 1.6 + \frac{2}{3} \times 2.4 = 3.2 \text{ m}$$

$$I_{xx} = \frac{bh^3}{36} = \frac{3 \times (2.4)^3}{36} = 1.152 \text{ m}^4$$

$$P = w \cdot A \cdot \bar{x} = 10 \times 3.6 \times 3.2 = 115.2 \text{ kN}$$



$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} = 3.2 + \frac{1.152}{3.6 \times 3.2}$$

= 3.8m from the free surface of the liquid

5) A triangular body of base width 3m & ht 2.4m is immersed vertically such that the base is at a depth of 1.6m from the free surface of the liquid spe wt of H<sub>2</sub>O = 10 kN/m<sup>3</sup>

$$\text{Area} = 3.6 \text{ m}^2$$

$$\bar{x} = 1.6 + \frac{2 \times 1.2}{3} = 2.4 \text{ m}$$

$$I_{xx} = \frac{bh^3}{36} = 1.152 \text{ m}^4$$

$$P = w \cdot A \cdot \bar{x} \quad \left[ \therefore \frac{8 \times (2.4)^3}{36 \times 10} \right] \\ = 10 \cdot 3.6 \cdot 2.4 \\ = 86.4 \text{ kN}$$

$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} = 2.4 + \frac{1.152}{3.6 \times 2.4} = 2.53 \text{ m}$$

= 2.53m from the free surface of the liquid

6) A square body of side 4m x 4m is immersed such that one of the diagonal is parallel to free surface & top is at a depth of 2m from the free surface find.

(a) total pressure

(b) centre of pressure

sol

$$\text{Area} = 16 \text{ m}^2$$

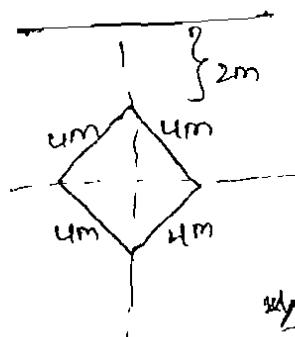
$$\bar{x} = 2 + \frac{4}{2}$$

$$= 4.83 \text{ m}$$

$$I_{xx} = \frac{a^4}{12} = \frac{4 \cdot 4 \cdot 4 \cdot 4}{12} = 21.33 \text{ m}^4$$

$$P = \rho \cdot g \cdot A \cdot \bar{x}$$

$$= 10 \cdot 10 \cdot 4.83 = 772.8 \text{ kN}$$



$$u^2 = 2x^2$$

$$x^2 = \frac{16}{2} = 8$$

$$x = 2.828$$

$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} = 4.83 + \frac{21.33}{16 \cdot (4.83)}$$

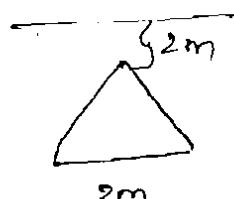
= 5.106 m from the free surface of the liquid

(7) An equilateral triangle of sides 3m is immersed vertically such that the top is at a depth of 2m from the free surface of the liquid calculate the total pressure & centre of pressure Assumed spe wt of  $H_2O = 10 \text{ kN}$

$$\text{Area} = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} \cdot 9$$

$$= 3.89 \text{ m}^2$$



$$h = \frac{\sqrt{3}}{2} \cdot a$$

$$\begin{aligned}
 \bar{x} &= 2 + \frac{2h}{3} \\
 &= 2 + \frac{2}{3} \cdot \frac{\sqrt{3}}{2} \cdot a \\
 \Rightarrow \bar{x} &= 2 + \sqrt{3} \\
 &= 2 + 1.732 \\
 &= 3.732 \text{ m}
 \end{aligned}$$

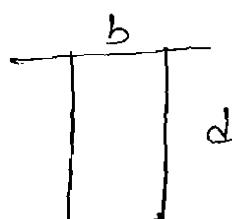
$$\begin{aligned}
 I_{xx} &= \frac{bh^3}{36} \\
 &= \frac{3.89 \left( \frac{\sqrt{3}}{2} \right)^3 a^3}{12} = \frac{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} a^3}{12} \\
 &= \frac{17.5}{12} = 1.45 \text{ m}^4
 \end{aligned}$$

$$\begin{aligned}
 P &= \omega \cdot A \cdot \bar{x} = 10 \cdot (3.89) (3.732) \\
 &= 145.09 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 h &= \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} = 3.732 + \frac{1.45}{3.89 (3.732)} \\
 &= 3.829 \text{ m } \text{ from the free surface of the liquid.}
 \end{aligned}$$

- 8) A rectangular body of breadth  $b$  and depth  $d$  is immersed vertically such that the top coincides with the free surface of the liquid. Find the position of centre of pressure.

$$\begin{aligned}
 h &= \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} \\
 &= \frac{d}{2} + \frac{\frac{bd^3}{6}}{\frac{b \cdot d - d}{2}} \\
 &= \frac{d}{2} + \frac{d}{6} \\
 &= \underline{\underline{d \left[ 1 + \frac{1}{3} \right]}}
 \end{aligned}$$

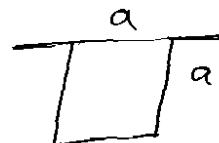


$$\bar{h} = \frac{d}{2} + \frac{4}{3} = \frac{2d}{3}$$

- 9) A square body of size  $a \times a$  is immersed vertically such that the top coincides with the free water surface locate the centre of pressure

Sol

$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}}$$

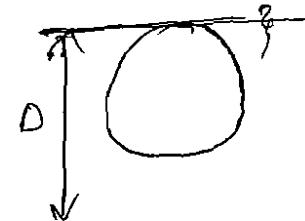


$$= \frac{a}{2} + \frac{\frac{a^4}{12} \times a}{\rho \cdot a^2 \cdot a}$$

$$= \frac{a}{2} + \frac{a}{6} = \frac{2a}{3}$$

- 10) A circular body of diameter "D" is immersed such that the top coincides with the free water surface locate the position of centre of pressure

$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}}$$



$$= \frac{D}{2} + \frac{\frac{\pi D}{64} \times D}{\frac{\pi D^2}{4} \times \frac{D}{2}}$$

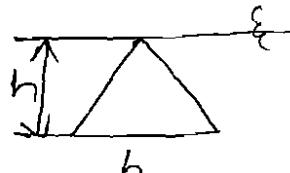
$$= \frac{D}{2} + \frac{D}{64} \times 8$$

$$= D \left[ \frac{4+1}{8} \right]$$

$$= \frac{5D}{8}$$

- 11) A triangular body base width  $b$  and height  $h$  is immersed vertically such that the open coincides with the free surface of the liquid calculate the centre of pressure

$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}}$$



$$= \frac{2}{3} \times h + \frac{\frac{b h^3}{36}}{\frac{1}{2} b h}$$

$$= \frac{2}{3}h + \frac{bh^3}{36} \times \frac{3}{bh^2}$$

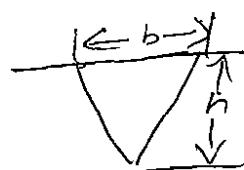
$$= \frac{2}{3}h + \frac{b}{12}$$

$$= h \left[ \frac{2}{3} + \frac{1}{12} \right] = h \left[ \frac{8+1}{12} \right]$$

$$= h \cdot \frac{9}{12} = \frac{3h}{4}$$

- 12) A triangle having base  $b$  and height  $h$  is immersed such that the base coincides with the free surface of the liquid. Locate the centre of pressure.

$$h = \bar{x} + \frac{\overline{fx}}{A \cdot \bar{x}}$$



$$= \frac{1}{3}h + \frac{bh^3}{36} \cdot \frac{1}{12 \cdot b \cdot h \cdot \frac{b}{3}}$$

$$= \frac{1}{3}h + \frac{b}{36} \times \frac{h}{6}$$

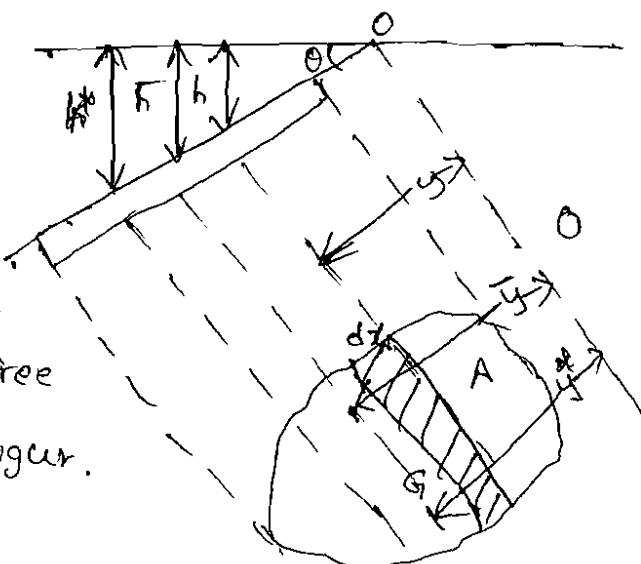
$$= \frac{1}{3} \left[ h + \frac{h}{2} \right] = \frac{h}{2}$$

Inclined surface submerged in liquid.

prove that centre of pressure always lies below centroid of the inclined surface.

Consider a plane surface of arbitrary shape immersed in liquid

in such a way that the plane of the surface makes an angle  $\theta$  with the free surface of the liquid as shown in Figur.



Ques

Let 'A' total area of inclined surface  $b$  = depth of small elementary strip from the free surface of the liquid the  $h$  = depth of centroid of inclined area from the free surface  $h^*$  = distance of centre of pressure from the surface of the liquid  $\theta$  = Angle made by the plane of the surface with the free surface of the liquid

Let the plane surface of produced meet the free liquid surface at  $\alpha$  then  $\alpha$  is the ~~ones~~ to the plane of the surface

Let  $y$  = distances of the elementary strip from the are  $\overline{\alpha}$   $y$  = distances of centroid of the inclined surface from the are  $\overline{\alpha}$   $y^*$  = distance of centre of pressure from the are  $\overline{\alpha}$

Consider a small elementary strip of area  $dA$  pressure force acting on the elementary strip,

Let

$dF$  = pressure force acting on the elementary strip

$$dF = \int dF$$

$$dF = P \times \text{Area} = (P.g.h) \times dA$$

$$= P.g.h \times b \cdot dy$$

$$dF = P.g.h \times b \cdot dy$$

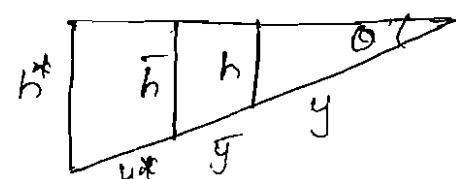
$$= P.g.b \cdot y \sin\theta dy$$

$$= P.g \cdot \sin\theta \int b \cdot dy \cdot y$$

$$= \dots \text{ (detailed steps)}$$

$\therefore$  Varignon's theorem

$$\overline{Ay} = \int dA \cdot h$$



$$\sin\theta = \frac{h^*}{g^*} = \frac{h}{y} = \frac{b}{y}$$

$$= P \cdot g \sin \theta A \cdot \bar{y} \quad [\because F = \bar{y} \sin \theta]$$

$$= P \cdot g A \bar{y} \sin \theta = P \cdot g \cdot A \bar{h}$$

$$F = (\text{specific weight } \omega = \rho g) \omega \cdot A \cdot h$$

$F = \omega \cdot \bar{h} \cdot A$  Total pressure acting on the body

$$F_x h^* = \int dF \cdot y$$

$\therefore \int y^2 dA = \text{moment}$

$$F_{xy^*} = \int dF \cdot y$$

$$= \int f \cdot g \cdot h \cdot dA \cdot y$$

$$I_O = I_{xx} + A\bar{y}^2$$

$$= \int f \cdot g \cdot y \sin \theta dA \cdot y$$

$$I_O = I_{xx} + A\bar{h}^2$$

$$= f \cdot g \sin \theta \int dA y^2$$

$$= f \cdot g \sin \theta I_O$$

$$= f \cdot g \sin \theta (I_{xx} + A\bar{g}^2)$$

$$= f \cdot g \sin \theta (I_{xx} + A\bar{g}^2)$$

$$F_{y^*} = f g \sin \theta (I_{xx} + A\bar{g}^2)$$

$$y^* = \frac{f g \sin \theta (I_{xx} + A\bar{g}^2)}{f g \sin \theta A \bar{y}}$$

$$y^* = \frac{I_{xx} + A\bar{g}^2}{A \bar{y}}$$

$$y^* = \bar{y} + \frac{I_{xx}}{A \bar{y}}$$

$$y^* = \frac{\bar{h}^*}{\sin \theta} = \bar{y} + \frac{I_{xx}}{A \bar{y}}$$

$$\Rightarrow \frac{h^*}{\sin \theta} = \frac{\bar{h}}{\sin \theta} + \frac{I_{xx}}{A \frac{\bar{h}}{\sin \theta}}$$

$$= \frac{1}{\sin \theta} \left( \bar{h} + \frac{I_{xx} \sin^2 \theta}{A \bar{h}} \right)$$

$$h^* = \bar{h} + \frac{I_{xx} \sin^2 \theta}{A \bar{h}}$$

If  $\theta = 90^\circ$  the above equation become same as that of

equation which is applicable to equation which applicable to vertically submerged surface for inclined surface,

total pressure  $F = w \cdot A \cdot h$

Centre of pressure  $\bar{h} = \bar{h} + \frac{I_{xx} \sin^2 \theta}{A \bar{h}}$

$$\boxed{\bar{h} = \bar{x}} \rightarrow \text{Note}$$

A rectangular body  $3m \times 2m$  is immersed such that the greatest and least depth are  $4m$  and  $2m$  respectively from the free surface of the liquid

Find

- 1) Total pressure
- 2) centre of pressure

Take specific wt of liquid  $10 \text{ kN/m}^3$

Ans

Sol

a) Total pressure force

$$= 10 \times (c_i \times 3) \times 3$$

$$= 360 \text{ kN}$$

(b)  $h = \frac{2+4}{2} = 3$

$$h' = 3 + \frac{\frac{3 \times 4 \sqrt{3}}{12} \sin 30^\circ}{3 \times 4 \times 3}$$

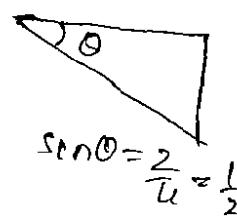
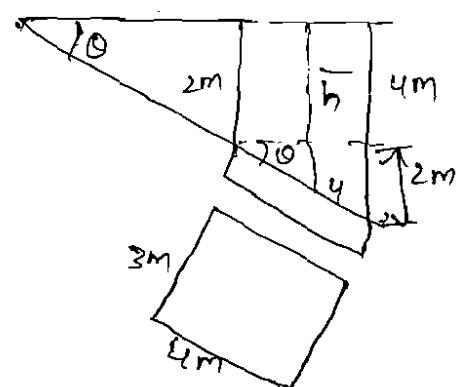
$$= 3 + \frac{\frac{3 \times 4 \sqrt{3}}{12} \times \left(\frac{1}{2}\right)^2}{3 \times 4 \times 3}$$

$$= 3 + \frac{1}{9}$$

$$= \frac{27+1}{9}$$

$$= \frac{28}{9}$$

$$= 3.11 \text{ m}$$



$$\sin \theta = \frac{2}{4} = \frac{1}{2}$$

$$\theta = 30^\circ$$

2) A square body of side  $4 \times 4$  is immersed such that the plane makes an angle of  $30^\circ$  with the free surface and top is at the depth  $2 \text{ m}$  from the free surface

Find

(a) Total pressure

(b) centre of pressure

Sol

Let  $H = 2+x$ .

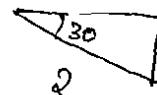
$$= 2 + (2 \cdot 2)$$

$$= 2(1 + \sin 30^\circ)$$

$\bar{h}$  = Initial depth ~~at centroid of~~ the inclined surface  $x \sin \theta$

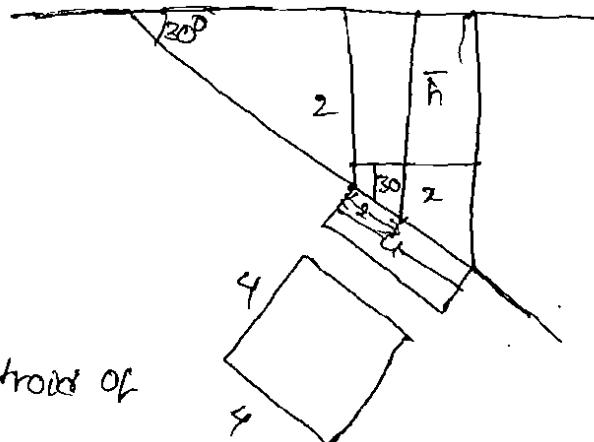
$$\bar{h} = 2(1 + \sin 30^\circ) = 3$$

$$F = 10 \times (4 \times 4) = 3 \\ = 480 \text{ kN}$$



$$\sin 30 = \frac{x}{2}$$
$$2 \sin 30 = x$$

$$h^* = \frac{3 + \frac{4}{12} \sin^2 30}{4 \times 4 \times 3}$$
$$= 3.11$$



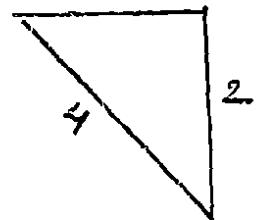
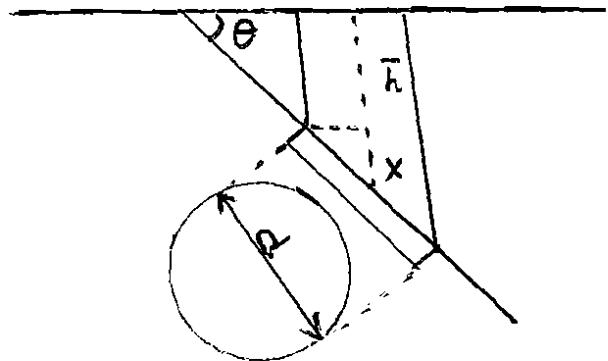
①

1. A circular body of diameter 4m is immersed such that the greatest & least depths are 3m & 1m. Find Total pressure and Centre of pressure.

$$\text{Sol: } \bar{h} = \frac{1+3}{2} = 2 \text{ m}$$

$$F = 10 \times \pi \times \frac{4^2}{4} \times 2 \\ = 251.327 \text{ kN}$$

$$\begin{aligned} \bar{h} &= 2 + \frac{\pi \times 4^4}{64} \sin^2 \theta \\ &\quad \frac{\pi \times 4^2}{4} \times 2 \\ &= 2 + 16/32 \times \frac{1}{4} \sin^2 \theta = 1/4 \\ &= 2 + \frac{1}{8} = \frac{17}{8} = 2.125 \text{ m.} \end{aligned}$$



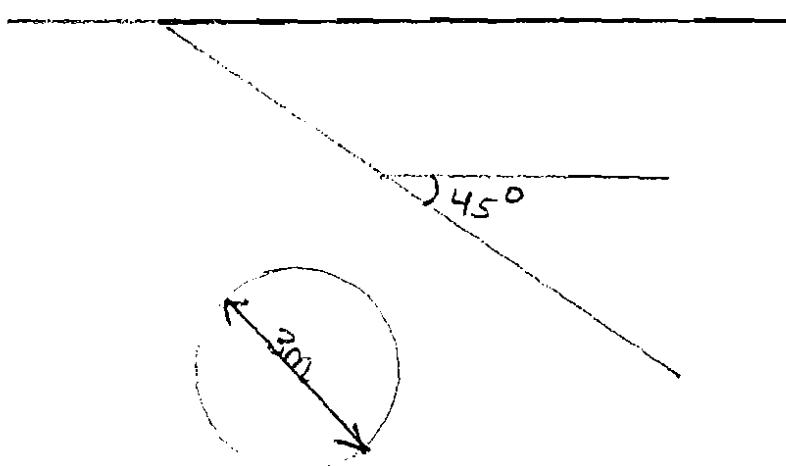
$$\sin \theta = 2/4,$$

$$\sin \theta = 1/2.$$

2. A circular 3m in diameter is immersed such that the top coincides with free surface of two liquid & make an angle  $45^\circ$  with free surface find total pressure Centre of pressure?

$$\text{Sol: } F = 10 \times \frac{\pi \times 3^2}{4} \times \frac{3}{2\sqrt{2}} \\ = 74.97 \text{ kN.}$$

$$\begin{aligned} \bar{h} &= 0 + \frac{3}{2} \times \sin 45^\circ \\ &= \frac{3}{2\sqrt{2}} = 1.06 \text{ m.} \end{aligned}$$



$$h^* = \frac{3}{2\sqrt{2}} + \frac{\pi \times 3^4}{64} \sin^2 45^\circ$$

$$\frac{\pi \times 3^2}{4} \times \frac{3}{2\sqrt{2}}$$

$$h^* = \frac{3}{2\sqrt{2}} + \frac{\frac{3}{8} \times \frac{1}{2}}{1/\sqrt{2}} = 1.325m$$

$$= 1.06 + \frac{3\sqrt{2}}{16} = 1.325m.$$

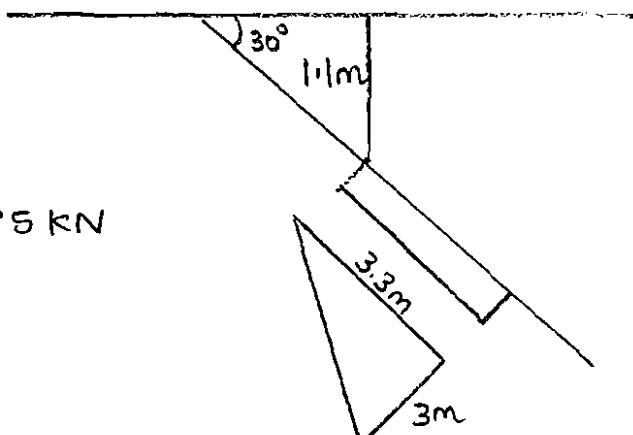
3. A Triangular body of base width 3m and height 3.3m is immersed such that the vertex is at a depth 1.2m from the free surface and plane makes an angle 30° with the free surface. Find a) Total pressure  
b) Centre of pressure.

Sof:  $\bar{h} = 1.2 + \frac{8 \times 3.3}{3} \sin 30^\circ = 2.3m$

$$F = 10 \times \frac{1}{2} \times 3.3 \times 3 \times 2.3 = 113.85 \text{ kN}$$

$$h^* = 2.3 + \frac{\frac{3 \times 3.3^3}{36} \times \sin^2 30^\circ}{\frac{1}{2} \times 3 \times 3.3 \times 2.3}$$

= 2.36m from the Free Surface of the liquid.



4. An equilateral  $\Delta^{le}$  3m x 3m is immersed such that the plane makes an angle  $30^\circ$  with the free surface is at depth of 1.6m from the free surface Find Total pressure and Centre of pressure.

$$\text{Sol: } \bar{h} = 1.6 + \frac{2.59}{3} \sin 30^\circ \\ = 2.03 \text{ m.}$$

$$A = \frac{\sqrt{3}}{4} \times 3^2 = 3.89 \text{ m}^2.$$

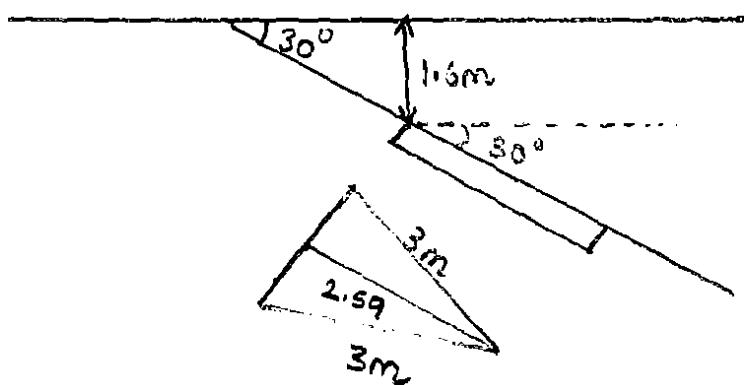
$$h = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 3 = 2.59 \text{ m.}$$

$$F = 10 \times 3.897 \times 2.03.$$

$$= 79.1 \text{ kN.}$$

$$h^* = 2.03 + \frac{3 \times (2.59)^3 \times \sin^2 30^\circ}{36} \\ = 2.03 + \frac{3 \times 17.5 \times 0.25}{36} \\ = 2.03 + 1.46 \\ = 3.49 \text{ m.}$$

$$= 2.07 \text{ from top.}$$



$$\sin 30^\circ = \frac{x}{0.86}$$

$$x = 0.43$$

$$\bar{h} = 1.6 + 0.43 \\ = 2.03 \text{ m.}$$



Curved surface submerged in liquid:-

Let us consider a curved surface AB submerged in static fluid as shown

Let "dA" be the area of small elementary strip at a depth of  $h$  from the free water surface.

The pressure intensity on the area dA is  $dP \rightarrow dA = \rho g h$ .

pressure force  $dF = (\rho g h) dA$

$$F = \int dF = \int \rho g h \cdot dA$$

pressure force  $dF = (\rho g h) dA$

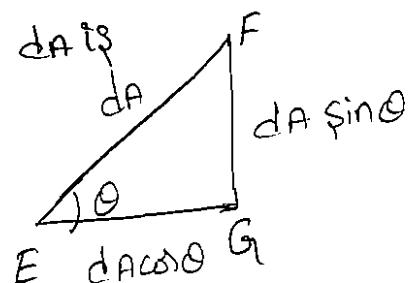
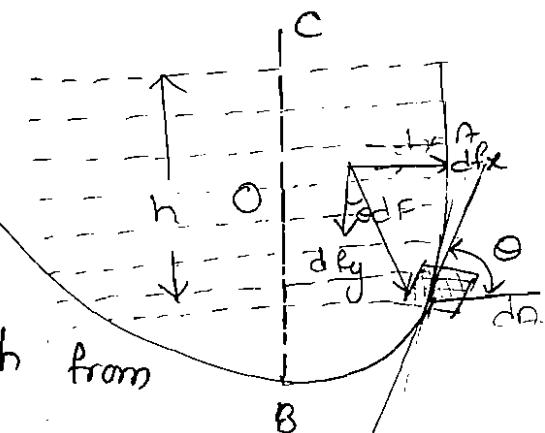
$$F = \int dF = \int \rho g h \cdot dA$$

here the direction of forces on the small areas are not in the same direction. It varies from point. Hence integration of equation of curved surface is impossible.

Let resultant force  $dF$  on the elementary strip makes an angle  $\theta$  with the vertical and tangent of the curved surface makes the same angle with the horizontal.

$$dF = \sqrt{dF_x^2 + dF_y^2}$$

$$F = \sqrt{F_x^2 + (F_y)^2}$$



$$dF_x = dF \sin\theta = \rho g h dA \sin\theta.$$

$$dF_y = dF \cos\theta = \rho g h dA \cos\theta.$$

Total force acting on the curved surface-

$$F_x = \int dF_x = \rho g \int h dA \sin\theta = \rho \cdot g \int h dA \sin\theta$$

$$F_y = \int dF_y = \rho \cdot g \int h dA \cos\theta.$$

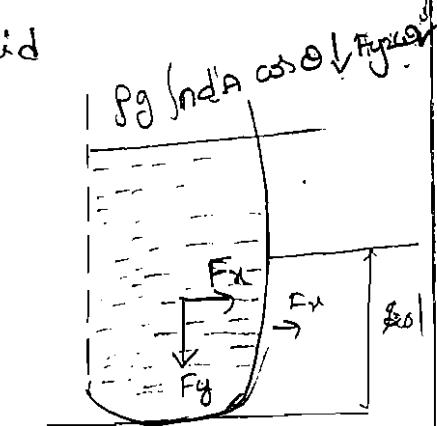
$\rho g \int h dA \sin\theta$  represents the total pressure force on the projected area of the curved surface on the vertical plane.

$$\rho g \int h dA \sin\theta \rightarrow F_x = \rho g \cdot A \cdot \bar{x}$$

$$\bar{h} = \bar{x} + \frac{\bar{I}_{xx}}{A \cdot \bar{x}}$$

$\rho g \int h dA \cos\theta$  represents the cut of the liquid supported by the curved surface up to free surface of the liquid.

$$\rho g \int h dA \cos\theta \rightarrow F_y = \omega V.$$



Note:-

In case figure curved surface AB is not supporting any liquid in such cases  $F_y$  = the cut of imaginary liquid supported by AB up to free surface of the liquid the direction of  $F_y$  will be taken in upward direction

$$F_x = \rho \cdot g \cdot A \cdot \bar{x} = \omega \cdot A \cdot \bar{x}$$

$$\bar{h} = \bar{x} + \frac{\bar{I}_{xx}}{A \cdot \bar{x}}$$

$F_y = \omega \times$  volume of the liquid supported by curved surface.

(1) Compute the horizontal and vertical components of the total force acting on a curved surface AB which is in the form of quadrant of a circle of radius 2 m & width as shown in fig. Take the width of the gate as unity.

$$F_x = \rho \cdot g \cdot A \cdot x$$

$$= \omega \cdot A \cdot x$$

$$= 1000 \times 9.81 \times (2 \times 1) \cdot \left(1.5 + \frac{2}{2}\right)$$

$$= 981 \times (2 \times 2.5)$$

$$= 49050 \text{ N}$$

$$= 49.05 \text{ kN.}$$

$$R = x + \frac{P_{xx}}{A \cdot x}$$

$$= 2.5 + \frac{1 \times 2}{(12) \cdot 1 \times 2 \times 2.5}$$

$$= 2.5 + \frac{1}{7.5}$$

$$= 2.83 \text{ m}$$

$$F_y = \omega \cdot v$$

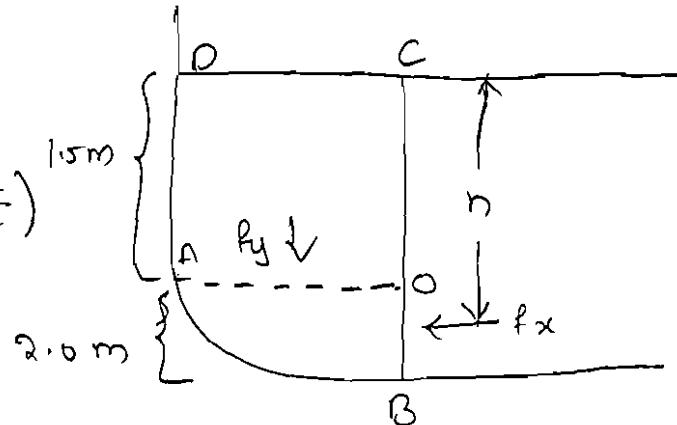
$$= 1000 \times 9.81 \times [1.5 \times 2.0 \times 1] + \left(\frac{\pi \times 2^2}{4} \times 1.0\right)$$

$$= 60.249 \text{ kN}$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\therefore F = \sqrt{(49.05)^2 + (60.249)^2}$$

$$\therefore F = 77.69 \text{ kN.}$$



(1) Fig shows a gate having a quadrant shape of radius 2m. find the resultant force due to water per unit length of the dam. Find also the angle at which the total force will act.

Sol:-

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\tan \theta = \frac{F_y}{F_x}$$

$$\theta = \tan^{-1} \left[ \frac{F_y}{F_x} \right]$$

$$F_x = w \cdot A \cdot x$$

$$= (1000 \times 9.81) \times (2 \times 1) \times \frac{x^2}{2}$$

$$= 19.62 \text{ kN.}$$

$F_y$  = wt. of the water supported by the curved surface  
(Imaginary)

$$= w \cdot V$$

$$= w \cdot A \cdot L$$

$$= 1000 \times 9.81 \times \frac{\pi \times 2^2}{4} \times 1$$

$$= 30.8 \text{ kN.}$$

$$\therefore F = \sqrt{F_x^2 + F_y^2}$$

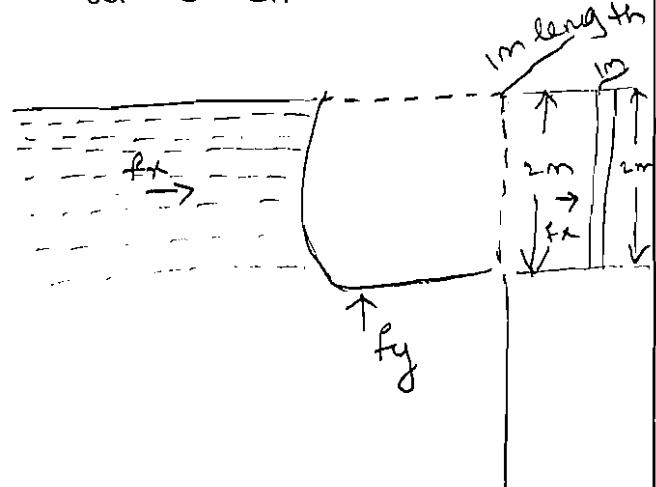
$$= \sqrt{(19.62)^2 + (30.8)^2}$$

$$= 36.52 \text{ kN.}$$

$$\tan \theta = \frac{30.8}{19.62}$$

$$\therefore \theta = \tan^{-1} \left[ \frac{30.8}{19.62} \right]$$

$$\therefore \theta = 57^\circ 30'$$



(2) Find magnitude and direction of the resultant force due to

water acting on a smaller side of cylindrical form with diameter when the gate is placed on the dam. In such a way that water is just go into spill take the length of the gate as 8m.

$$\text{So! } F = \sqrt{F_x^2 + F_y^2}$$

$$\tan\theta = \frac{F_y}{F_x}$$

$$\theta = \tan^{-1} \left[ \frac{F_y}{F_x} \right]$$

$$F_{x2} = \omega \cdot A \cdot \bar{x}$$

$$= (1000 \times 9.81) (4 \times 8) \frac{4}{2}$$

$$= 627.8 \text{ kN}$$

$F_y$  = coh. of water supported by the curved surface  
(Imaginary)

$$= \omega \times V$$

$$= (1000 \times 9.81) \frac{\pi \times 2^2}{L} \times 8$$

$$= 493.10 \text{ kN}$$

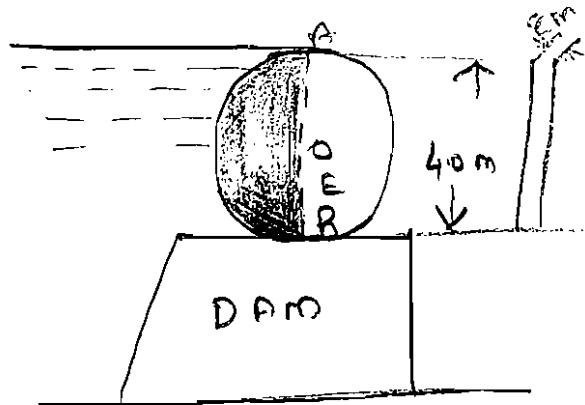
$$\therefore F = \sqrt{(627.8)^2 + (493.10)^2}$$

$$= 798.32 \text{ kN}$$

$$\therefore \theta = \tan^{-1} \frac{493.10}{627.8}$$

$$\therefore \theta = 38^\circ 81'$$

- ③ Find the horizontal and vertical component of center pressure acting on the face of a fainter gate of  $90^\circ$  sector of radius um as shown in figure.  
Take the width of the gate as unity.



$$\text{Soln: } F = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1} \frac{F_y}{F_x}$$

$$\therefore F_x = \omega \cdot A \cdot g$$

$$= (1000 \times 9.81) (5.65 \times 1) \times \frac{5.65}{2}$$

$$\approx 156.91 \text{ kN.}$$

$$F_y = 1000 \times 9.81 \times 2 \cdot \left[ \frac{y^{3/2}}{3/2} \right]_0^{12}$$

$$= 1000 \times 9.81 \times \frac{4}{3} 12^{3/2}$$

$$= 1000 \times 9.81 \times \frac{4}{3} \sqrt{12} \cdot 12$$

$$= 1000 \times 9.81 \times 16\sqrt{2}$$

$$F_y = 543.725 \text{ kN.}$$

$$\therefore F = \sqrt{(156.9)^2 + (543.7)^2}$$

$$\therefore F = 891.3 \text{ kN.}$$

$$\therefore \theta = \tan^{-1} \left[ \frac{543.7}{156.9} \right]$$

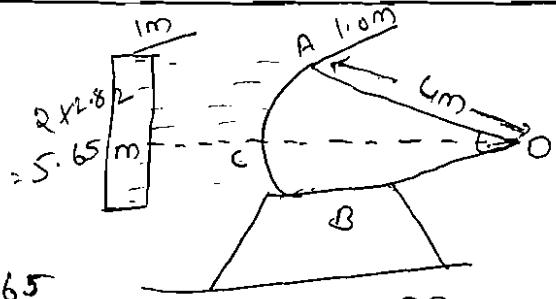
$$\therefore \theta = 37^\circ 35'.$$

### No (BUOYANCY)

Whenever a body is placed over a liquid either it sinks down or floats on a liquid. If we analyse the phenomena of floating we find that the body placed over a liquid is subjected to the following two forces

(1) Gravitational force and

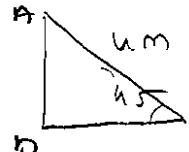
(2) up thrust of the liquid.) No



$$\sin \alpha = \frac{AD}{AC}$$

$$AB = AC \sin 45^\circ$$

$$\approx 282 \text{ m}$$



Gravitational force:



up thrust of liquid.

FLUID KINEMATICS

fluid kinematics is the branch of fluid mechanics which studies the fluid motion without considering the force causing the motion.

The methods used to study the fluid motion are:

- 1) Lagrangian Method and
- 2) Eulerian Method.

### 1) Lagrangian method:-

In this method a single fluid particle is followed during its motion and its velocity, acceleration and density etc., are described.

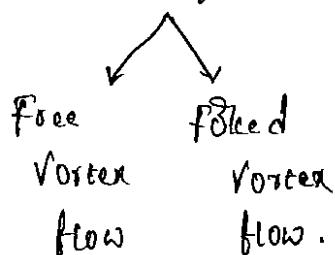
### 2) Eulerian method :-

It deals with the study of flow pattern of all particles simultaneously at one section. In this method the path traced by all the particles at one section and one-time are studied in detail.

### \* Classification of fluid flow:-

The fluid flow is classified as:

- i) Steady - unsteady flow
- ii) uniform and non-uniform flows
- iii) Laminar and Turbulent flows
- iv) compressible and incompressible flows
- v) Rotational and irrotational flow
- vi) 1-D, 2-D and 3-D flow
- vii) Vortex flow



i) Steady flow :- If discharge is constant at every point along the path of the flow. the flow is treated as steady flow.

(Q1)

The type of flow in which fluid characteristics like velocity, pressure, density etc. at a point does not change with time.

Mathematically it can be expressed as

$$\frac{\partial V}{\partial t} = 0 \quad \frac{\partial P}{\partial t} = 0 \quad \& \quad \frac{\partial \rho}{\partial t} = 0$$

↓            ↓            ↓  
Velocity      pressure      density.

Ex:- flow through constant diameter pipe, flow through tapering pipe.

\* unsteady flow :- If discharge is not constant at every point along the path of the flow. the flow is treated as unsteady flow.

(Q2)

The type of flow in which fluid characteristics like velocity, pressure, density etc., at a point changes with respect to time.

Ex:- flow through channels during floods, flow through channels when the gates are just lifted.

ii) uniform flow :- If velocity is constant at every point along the path of the flow. the flow is treated as uniform flow.

Ex:- flow through open channel with constant depth, flow through pipe with constant diameter.

\* Non-uniform flow :- If velocity is not constant at every point along the path of the flow. the flow is treated as non-uniform flow.

Ex:- flow through tapering pipe, flow through channels during floods.

### iii) Laminar flow:-

The flow in which liquid particles move in layers such that one layer slides over the other layer.

The path lines of fluid particles are straight lines and they are parallel to each other.

No fluid particles will cross each other in laminar flow.

Laminar flow is observed in the case of highly viscous fluids.

Ex:- flow of thick oil in a small tube, flow of blood through the veins of human body.

### \* Turbulent flow:-

It is the type of flow in which the path lines of fluid particles are irregular and crossing each other.

The flow is erratic (unpredictable = the direction of the fluid particles cannot be identified)

Ex:- flow through river during floods, flow through open channels when the gates are just lifted.

### ii) Compressible flow:-

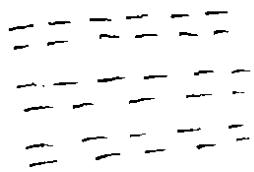
If the density of the fluid particles changes from point to point along the path of the flow, the flow is (treated) called compressible flow.

Ex:- flow of gas through the nozzle, flow of gas through the gas turbines

### \* Incompressible flow:-

The type of flow in which the density of fluid particles does not change. (Density constant) from point to point is called incompressible flow.

Ex:- pipe flow and channel flow.



i) Rotational flow :- If fluid particles rotate about its own mass centre while flowing. the flow is treated as rotational flow.

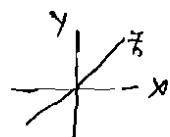
Ex:- flow inside the casing of a centrifugal pump, flow of tidal waves.

\* Irrational flow :- If the fluid particles does not each rotate about its own mass centre while flowing. the flow is treated as irrational flow.

Ex:- all flow through the pipes and open channels are irrational flows.

ii) 1-D, 2-D and 3-D flow :-

1-D flow :- the type of flow in which flow lines are represented by a straight line is known as one-dimensional flow.



(Q1)

The type of flow in which the flow parameters such as velocity is a function of time and one space coordinate only.

2-D flow :- The type of flow in which stream lines or flow lines represented by a curved line is known as 2-D flow

(Q1)

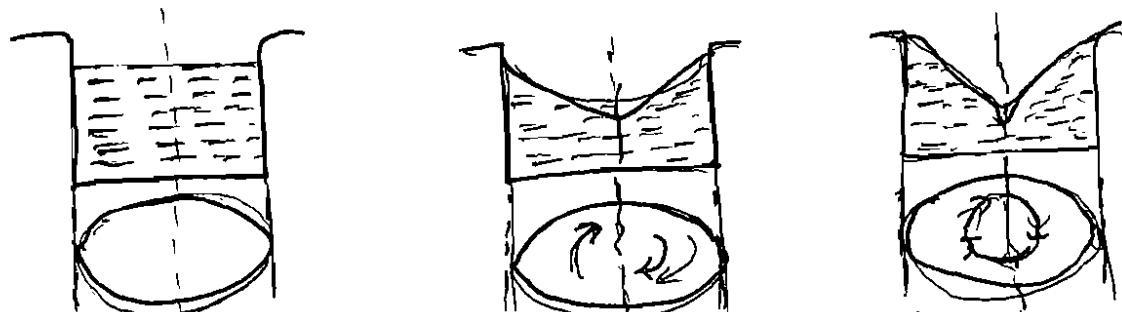
The type of flow in which the fluid parameters such as velocity is a function of time and two space coordinate and other coordinates are neglected.

3-D flow :- the type of flow in which stream lines may be represented in space along 3 mutual per direction is known as 3-D flow

(Q1)

The type of flow in which the fluid parameters such as velocity is a function of time and the three space coordinate.

## Vortex flows



Let us consider a cylindrical vessel containing some liquid and start rotating it about its vertical axis. We see that the liquid will also start revolving along with the vessel. After sometime we shall see that the liquid surface no longer remains level but it has been depressed down at the axis of the rotation and has risen up near the walls of vessel on all sides. This type of flow in which the liquid flows continuously round a curved path about a fixed axis of rotation is called Vortex flow.

### 1) Forced Vortex flow

The type of Vortex flow in which the External torque is required to rotate the fluid mass is known as forced Vortex flow.  
Ex:- flow of liquid inside the impeller of a centrifugal pump, flow of water through the runner of a turbine, in vertical cylinder containing liquid which is rotated about its central axis with a constant angular velocity.

### 2) Free Vortex flow

The type of Vortex flow in which no External torque is required to rotate the fluid is known as free Vortex flow.

Ex:- A whirlpool in a river (consider flow of a liquid around a circular bend in a pipe), flow of liquid through a hole provided at the bottom of a container.

Classify the following types of flows-

- 1) flow through constant diameter pipe (steady uniform flow)
- 2) flow through tapering pipe (steady non-uniform flow)
- \*\* 3) flow through capillary tube (Laminar flow)
- \*\* 4) flow of blood in veins of human beings (Laminar flow)
- 5) flow through open channels during floods (unsteady non-uniform flow/turbulent flow)
- 6) flow of Kerosene through the wicks of the stove (uniform flow)
- 7) flow of water through the fountains (unsteady non-uniform flow)
- 8) flow of tidal waves (rotational flows)
- 9) flow in a pipe considered as 1-D flow.
- 10) flow between parallel plates of infinite extent & known as 2-D flow.
- 11) The flow in a main stream & diverging pipe or channel is 3-D flow.
- 12) the flow in converging/diverging pipe is called as 3-D flow.
- 13) the flow in a prismatic open channel in which the width of the water depth are in the same order of magnitude is considered as 3-D flow.
- 14) motion of a liquid in a rotating tank is called rotational flow.
- 15) flow of water in a wash basin is known as irrotational flow.

## Classification of Flow Lines:

→ The fluid motion can be described in terms of path line, stream line, streak line

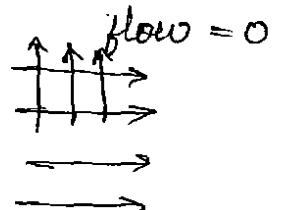
### Path line :-

A path line is a curve traced by a single fluid particle during its motion. This is the outcome of Lagrangian approach.

### Stream line :-

It is the path traced by number of fluid particles is known as stream line. It is the outcome of Eulerian approach.

### Characteristics Of Stream line :-

- 1) The flow across a stream line is zero 
- 2) Two stream lines won't cross each other
- 3) Stream lines are parallel
- 4) It is steady uniform flow
- 5) If stream lines are converging it indicates the velocity is increasing (accelerating flow) 
- 6) If stream lines are diverging it indicates retarding flow i.e. velocity is decreasing in the direction of the flow

### Streak line :-

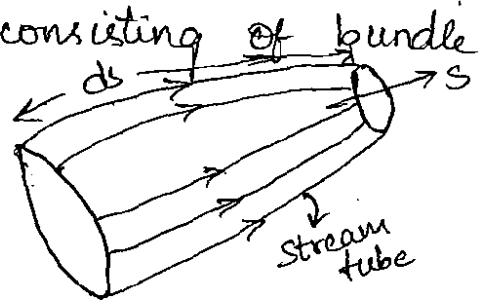
It is defined as a line i.e. traced by a fluid particle passing through a fixed point in a flow field 

Ex:- Smoke in the case of gases, smoke coming from a cigarette.

\*\*\* In the case of steady flow since there is no change in the fluid pattern stream line is same as that of streak line. In other words, for steady flow stream line, streak line and path line are identical

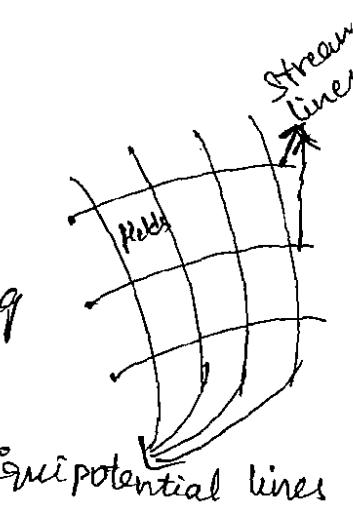
## Stream Tube :

A stream tube is an imaginary tube consisting of a bundle of stream lines passing through a small closed curve. A stream tube is completely bounded by stream lines at all sides except at ends. The flow across the stream tube is zero. The fluid can enter or leave the stream tube only at ends. The concept of stream tubes extremely used in solving fluid problems.



## \* \* Flow Net :-

A mesh or net formed by number of stream lines and equipotential lines intersecting orthogonally (perp.)



## Stream lines (flow lines)

The line along which flow takes place is known as stream line (or) flow line

Water flows from points of high heads to points of low heads and makes smooth curves representing the paths followed by moving particles of water. These lines are called flow lines / stream lines

## Equipotential lines :-

A line joining all the points of having equal total heads (or) potential head is called equipotential line

$$\text{Total head} = z + \frac{V^2}{2g} + \frac{P}{\rho g}$$

## \* \* characteristics of flow net (or) properties of flow net :-

- 1) flow lines & equipotential lines are orthogonal to each other
- 2) Two flow lines /two equipotential lines can never meet (i) cross each other
- 3) fields are kept approximately squares

## Applications :

- 1) For the given boundaries of flow if the velocity and pressure distribution at a section are known the velocity and pressure distribution at any other section can be calculated

$$A_1 V_1 = A_2 V_2$$

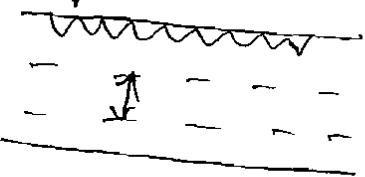
- 2) Loss of flow due to seepage in earthen dams and unlined canals can be determined
- 3) Uplift pressure under a dam or any hydraulic structure can be determined

## Assumptions involved in flow net :-

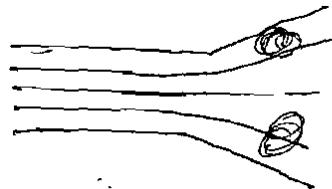
- 1) Flow is steady
- 2) Flow should be irrotational
- 3) Flow is governed by gravitational forces
- 4) The fluid weight does not control the flow phenomenon

## Limitations of flow net :-

- 1) Flow net Analysis is not applicable close to the boundaries, because the effect of viscosity are permanent



- 2) Flow net analysis is not applicable for sharp diverging flows because the flow pattern is not represented by flow net

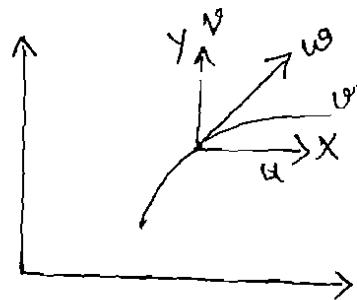


## Velocity and Acceleration :

Let  $v$  be the resultant velocity at any point in a fluid flow.

Let  $u, v, w$  are its components in  $x, y$  and  $z$  directions.

The velocity components are the functions of space co-ordinate and Time.



Mathematically it can be expressed as

$$u = f_1(x, y, z, t)$$

$$v = f_2(x, y, z, t)$$

$$w = f_3(x, y, z, t)$$

Resultant velocity is given by

$$v = \sqrt{u^2 + v^2 + w^2}$$

Let  $a_x, a_y$  and  $a_z$  are total acceleration in  $x, y$  and  $z$  directions respectively. Then by the chain rule of differentiation, we have

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial u}{\partial t}$$

$$\therefore a_x = \frac{\partial u}{\partial x} \cdot u + \frac{\partial u}{\partial y} \cdot v + \frac{\partial u}{\partial z} \cdot w + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dv}{dt} = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

For Steady flow

$$\frac{\partial u}{\partial t} = 0 ; \quad \frac{\partial v}{\partial t} = 0 , \quad \frac{\partial w}{\partial t} = 0 .$$

Hence accelerations in  $x, y$  and  $z$  directions becomes

$$a_x = \frac{\partial u}{\partial x} \cdot u + \frac{\partial u}{\partial y} \cdot v + \frac{\partial u}{\partial z} \cdot w$$

$$a_y = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} \quad \text{and}$$

$$a_z = u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z}$$

Total acceleration is  $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Local acceleration :

It is defined as the rate of change of velocity with respect to time at a given point in a flow field is known as local acceleration.  $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}$  and  $\frac{\partial w}{\partial t}$  are local accelerations.

Convective acceleration :

It is defined as the rate of change of velocity due to change of position of fluid particles in a fluid flow.

In the above equations the expression

$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$  is called local accelerations.

In the above equations the expression other than  $\frac{\partial u}{\partial t}$ ,  $\frac{\partial v}{\partial t}$  and  $\frac{\partial w}{\partial t}$  is called Convective acceleration.

Acceleration in any direction = Convective acceleration + local acceleration.

For Steady flow: Total acceleration = Convective acceleration.

For Uniform flow: Total acceleration = local acceleration

For Steady - uniform flow :

Total acceleration = 0

### problems

1) The velocity vector in a fluid flow is given by

$v = 4x^3 i - 10xy^2 j + 2t k$ . Find the velocity and acceleration of a fluid particle at (2, 1, 3).

Sol:-

Given

$$v = \begin{matrix} 4x^3 \\ u \\ v \\ w \end{matrix} i - 10xy^2 j + 2t k$$

$$u = 4x^3 \quad v = -10xy^2 \quad w = 2t$$

$$u = 4 \times 2^3 \quad = -10 \cdot 2 \cdot 1 \quad = 2 \times 1$$

$$u = 32 \quad v = -40 \quad w = 2$$

$$v = \sqrt{32^2 + (-40)^2 + 2^2} = 51.26$$

$$v = 4x^3$$

$$\frac{\partial v}{\partial x} = 12x^2 \quad \frac{\partial v}{\partial y} = 0 \quad \frac{\partial v}{\partial z} = 0 \quad \frac{\partial v}{\partial t} = 0$$

$$a_x = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= 32 \times 12x^2$$

$$= 32 \times 12 \times 4$$

$$a_x = 1536$$

$$v = -10xy$$

$$\frac{\partial v}{\partial x} = -20xy \quad \frac{\partial v}{\partial y} = -10x^2 \quad \frac{\partial v}{\partial z} = \frac{\partial v}{\partial t} = 0$$

$$a_y = 32 \times (-20xy) + (-40) \times -10x^2$$

$$= -32(20)(2) + 400 \cdot 4$$

$$= 320 \text{ units.}$$

$$w = 2t$$

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial t} = 2$$

$$a_z = 2$$

$$\therefore a = \sqrt{(1536)^2 + (320)^2 + 2^2} = 1568.9$$

2. A fluid flow field is given by  $\mathbf{v} = x^2y\mathbf{i} + y^2z\mathbf{j} - (2xyz + yz^2)\mathbf{k}$ .  
 Prove that it is a case of possible steady incompressible flow. Calculate the velocity and acceleration at a point (2, 1, 3)

Sol:-

Given

$$\mathbf{v} = x^2y\mathbf{i} + y^2z\mathbf{j} - (2xyz + yz^2)\mathbf{k}$$

$$u \quad v \quad w$$

$$u = x^2y \quad v = y^2z \quad w = -(2xyz + yz^2)$$

$$u = 4 \quad v = 3 \quad w = -(12 + 9) = -21$$

$$\therefore |v| = \sqrt{4^2 + 3^2 + (-21)^2}$$

$$= 21.587$$

$$u = x^2y$$

$$\frac{\partial u}{\partial x} = 2xy \quad \frac{\partial u}{\partial y} = x^2 \quad \frac{\partial u}{\partial z} = 0 \quad \frac{\partial u}{\partial t} = 0$$

$$\therefore a_x = 2xy \cdot (x^2y) + 3(x^2)$$

$$= 4 \cdot 4 + 3 \cdot 4$$

$$= 16 + 12$$

$$= 28$$

$$v = y^2z$$

$$\frac{\partial v}{\partial y} = 2yz \quad \frac{\partial v}{\partial z} = y^2 \quad \frac{\partial v}{\partial x} = 0 \quad \frac{\partial v}{\partial t} = 0$$

$$\alpha_y = (3)2yz - 21(y^2)$$

$$= 18 - 21$$

$$= -3$$

$$\omega = -(2xyz + yz^2) \quad \alpha_z = -9$$

$$\frac{\partial \omega}{\partial x} = -2yz \quad \frac{\partial \omega}{\partial y} = -(2xz + z^2) \quad \frac{\partial \omega}{\partial z} = 2xy + 2yz$$

$$\frac{\partial \omega}{\partial t} = 0.$$

$$\alpha_z = 4(-2yz) + (-3)(2xz + z^2) + 21(2xy + 2yz)$$

$$= 4(-6) + (-3)(12+9) + 21(4+6)$$

$$= -24 - 63 + 21(10)$$

$$\alpha_z = 123$$

$$\therefore \alpha = \sqrt{(28)^2 + 9 + (123)^2}$$

$$\alpha = 126.18$$

## Continuity equation:-

Like solid mechanics in fluid mechanics also the following three principles are used for the analysis of fluid motion. They are :-

- 1) Conservation of mass
- 2) Conservation of energy
- 3) Conservation of momentum.

### 1) Conservation of mass:-

It states that mass can neither be created nor be destroyed on the basis of this principle the continuity equation is derived.

### 2) Conservation of energy:-

It states that energy can neither be created nor destroyed. Based on this principle energy equation can be derived.

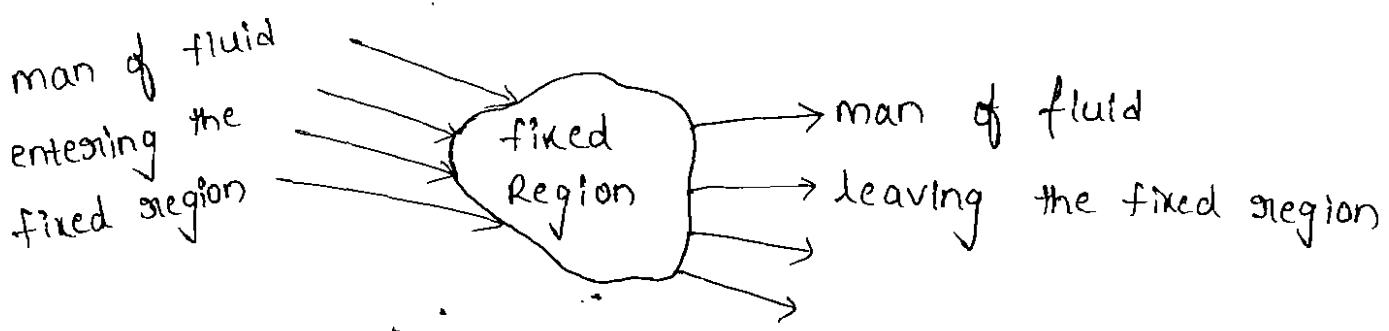
### 3) Conservation of momentum:-

The resultant impulse (force $\times$ time) acting on the fluid is equal to change in momentum of the fluid.

Based on this principle momentum equation is derived.



In fm the conservation of mass mathematically expressed as



Rate of increase of fluid mass within the fixed region

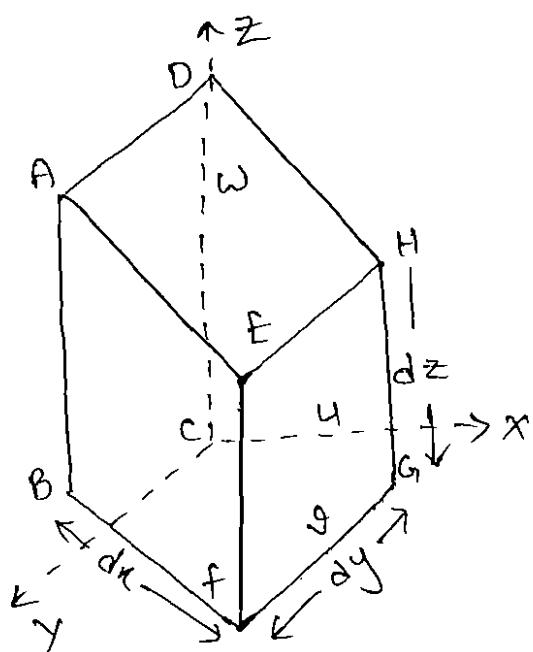
= mass of fluid leaving the fixed region

Region - mass of fluid entering the fixed region

for steady flow: The rate of increase of fluid mass within the fixed region = 0

Hence, the rate at which the fluid mass enters the fixed region = The rate at which the fluid mass leaves the fixed region.

continuity equation for 3-D flow:-



Consider a fluid element of length  $dx$ ,  $dy$  and  $dz$  in the direction of  $x, y$  and  $z$ . Let  $u, v, w$  are inlet velocity components in  $x, y$  and  $z$  directions.

mass of the fluid entering the face ABCD is =

$$\begin{aligned} &= \text{Density} \times \text{volume of fluid} (A \times h) \\ &= \rho \times Q \\ &= \rho \times A \times V && (\text{Volume} = \text{discharge}) \\ &= \rho \times (dz \cdot dy) \times u \end{aligned}$$

Change of fluid mass per unit distance =

$$= \frac{\partial}{\partial x} (\rho u \cdot dz \cdot dy) \cdot dx$$

mass of fluid leaving the face EFGH per second is

$$= (\rho \cdot dz \cdot dy \cdot u) + \frac{\partial}{\partial x} (\rho \cdot u \cdot dz \cdot dy) \cdot dx$$

Net gain in fluid mass in  $x$ -direction =  $-EFGH + ABCD$

$$= -\frac{\partial}{\partial x} (\rho \cdot u \cdot dz \cdot dy) \cdot dx$$

Net gain in mass in  $y$ -direction =  $-\frac{\partial}{\partial y} (\rho \cdot v \cdot dx \cdot dy) \cdot dz$

Net gain in mass in  $z$ -direction =  $-\frac{\partial}{\partial z} (\rho \cdot w \cdot dx \cdot dy) \cdot dz$

Total net gain in mass of fluid =  $-\left(\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w)\right) \cdot dx \cdot dy \cdot dz$

Rate of increase of mass of fluid elements with respect to time =  $\frac{\partial}{\partial t} (\rho \cdot dx \cdot dy \cdot dz)$

According to principle of conservation of mass the rate of increase of mass of fluid element with respect to time = difference of fluid mass entering the fixed region and leaving the fixed region.

$$\frac{\partial}{\partial t} (P, dx, dy, dz) = - \left[ \frac{\partial}{\partial x} (PV) + \frac{\partial}{\partial y} (PW) + \frac{\partial}{\partial z} (PU) \right] dx \cdot dy \cdot dz$$

$$\frac{\partial P}{\partial t} = - P \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right]$$

$$\frac{\partial P}{\partial t} + \left( \frac{\partial Pu}{\partial x} + \frac{\partial Pv}{\partial y} + \frac{\partial Pw}{\partial z} \right) = 0$$

This is a general continuity equation.

case(i):- If flow is steady continuity equation becomes

$$\left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0$$

case(ii):- for steady incompressible flow  
 ↓  
 (density constant)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

case(iii):- for 2-D flow i.e., only in x and y directions

for steady - incompressible flow

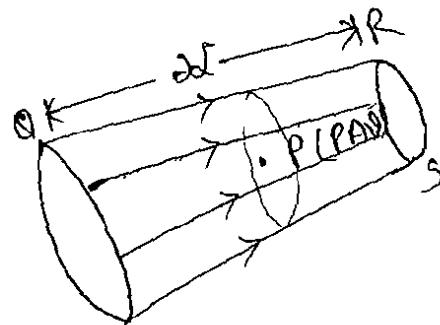
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

## Continuity equation in 1-D flow

Let us consider an

elementary stream tube of

length  $ds$  as shown in figure.



let us consider a point  $P$  at

the centre of the tube let the mass density, velocity and area at the section be  $\rho$ ,  $v$  and  $A$  respectively.

The mass rate of flow exactly at  $P = P \times A$

$$= \rho \times A \times v$$

The mass rate of flow entering the tube through

$$\text{the face } TQ \text{ (m)} = \rho A v - \frac{d}{ds} (\rho A v) \frac{ds}{2}$$

The mass rate of flow leaving the tube through the

$$\text{face } RS \text{ (m)}_2 = \rho A v + \frac{d}{ds} (\rho A v) \frac{ds}{2}$$

Net gain in mass rate of flow  $= m_1 - m_2$

$$= 1 - \frac{d}{ds} (\rho A v) ds - ①$$

$$\text{Rate of acc of mass with respect to time} = \frac{d}{dt} (P \cdot A \cdot ds) - ②$$

According to law of conservation of mass ① and ②

are equal. On equating them, we get

$$\frac{d}{dt} (P \cdot A \cdot \frac{ds}{dt}) = - \frac{d}{ds} (P \cdot A) ds$$

$$\therefore \frac{\partial PA}{\partial t} + \frac{\partial}{\partial x} (PAv) = 0$$

$\therefore$  It is the general continuity equation for 1-D flow.

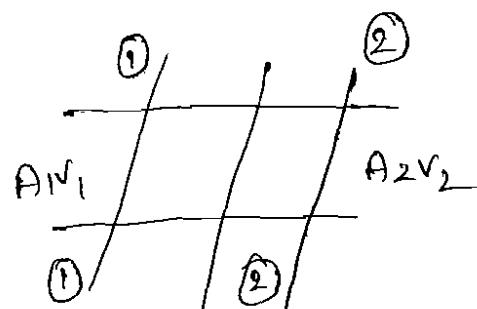
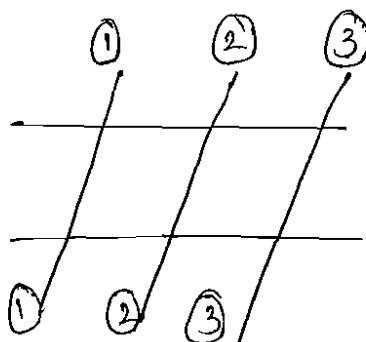
Case (i) :- for steady flow the continuity equation becomes  $\frac{\partial}{\partial x} (PAv) = 0$

Case (ii) :- for steady - incompressible flow the continuity equation becomes  $\frac{\partial}{\partial x} (Av) = 0$

from the above equation the equation can be written as

$$A_1v_1 = A_2v_2 = A_3v_3, \text{ the subscripts } 1, 2, 3$$

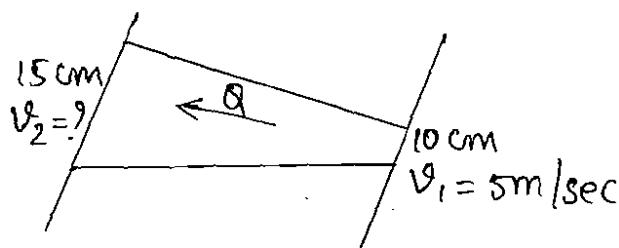
represents cross sectional areas and velocity of fluid at three different sections.



$$Av = 0$$

problems:

- 1) The diameter of a pipe at the sections ① and ② are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section ① is 5m/sec. Determine also the velocity at section ②.



$$Q = A \times V$$

$$Q = \frac{\pi \times \left(\frac{10}{100}\right)^2}{4} \times 5 = 0.039 \text{ m}^3/\text{sec}$$

$$Q = A_2 V_2$$

$$0.039 = \frac{\pi \times \left(\frac{15}{100}\right)^2}{4} \times V_2$$

$$\therefore V_2 = 2.22 \text{ m/sec.}$$

- 2) A 30 cm diameter pipe conveying water branches into two pipes of diameters 20cm and 15cm respectively. If the average velocity in the 30cm diameter pipe is 2.5 m/sec. Find the discharge in this pipe also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2m/sec.

Sol:

$$Q = \frac{\pi \times \left(\frac{30}{100}\right)^2 \times 2.5}{4} \quad (\text{A} \times \text{V})$$

$$= 0.1767 \text{ m}^3/\text{Sec}$$

$$Q = Q_2 + Q_3$$

$$0.1767 = \frac{\pi \times \left(\frac{20}{100}\right)^2 \cdot 2}{4} + \frac{\pi \times \left(\frac{15}{100}\right)^2 \times V_3}{4} \quad (\text{A}_2 V_2) \quad (\text{A}_3 V_3)$$

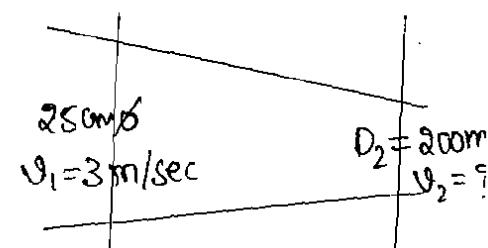
$$\therefore V_3 = 6.44 \text{ m/sec.}$$

- 3) A 25cm diameter pipe carries oil of specific gravity 0.9 at a velocity of 3m/sec at another section the diameter is 20cm. Find the velocity at this section and also the mass rate of flow of oil.

Sol:

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} \times \left(\frac{25}{100}\right)^2 \times 3 = \frac{\pi}{4} \times \left(\frac{20}{100}\right)^2 \times V_2$$



$$\therefore V_2 = 4.68 \text{ m/sec.}$$

$$Q = A_1 V_1 = A_2 V_2$$

$$\text{Mass rate of flow} = P \times Q$$

$$= P \times A_1 V_1$$

$$= (0.9 \times 1000) \times \frac{\pi}{4} \times \left(\frac{25}{100}\right)$$

$$= 132.23 \text{ kg/sec.}$$

Q) A Jet of water from a 25mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy. what will be the diameter at a point 4.5m above the nozzle. If the velocity with which the jet leaves the nozzle is 12 m/sec

Sol:

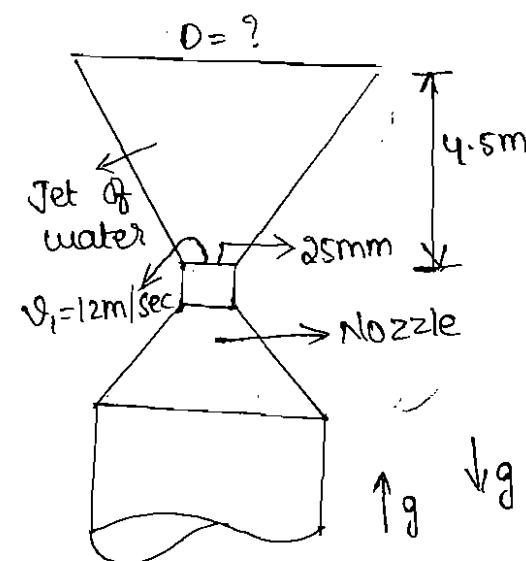
$$V^2 - U^2 = \pm 2gh$$

$$V^2 - U^2 = - 2gh$$

$$V_2^2 - V_1^2 = - 2gh$$

$$V_2^2 = - 2 \times 9.81 \times 4.5 + (12)^2$$

$$\therefore V_2 = 7.46 \text{ m/sec}$$



Continuity equation is  $A_2 V_2 = A_1 V_1$

$$\frac{\pi}{4} \cdot D_2^2 \times 7.46 = \frac{\pi}{4} \times \left(\frac{25}{1000}\right)^2 \times 12$$

$$D_2 = 31.7 \text{ mm}$$

Velocity potential function:

It is defined as scalar function of space and time such that its partial -ve derivative w.r.t any direction gives the fluid velocity in that direction.

Mathematically it is defined as -

$u, v, w$  are velocity components in  $x, y$  and  $z$  respectively.

$$\therefore u = -\frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \phi}{\partial y}$$

$$w = -\frac{\partial \phi}{\partial z}$$

The -ve sign indicates that the velocity potential increases in the case of flow.

In other words it indicates that the flow is always in the direction of flowing pipe or  $\downarrow$

For steady-incompressible 3D flow the continuity eqn is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Substituting  $u, v, w$  values in above eqn, we get

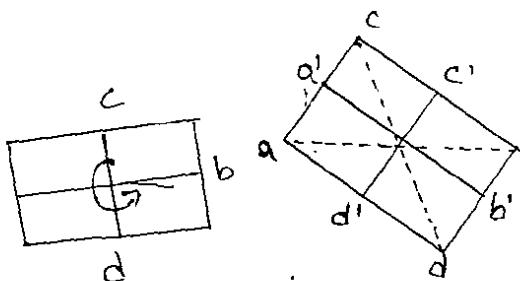
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

The above eqn is called as Laplace equation for any function  $\phi$  that satisfies Laplace eqn will correspond to some case of fluid flow (it may be rotational / irrotational flow)

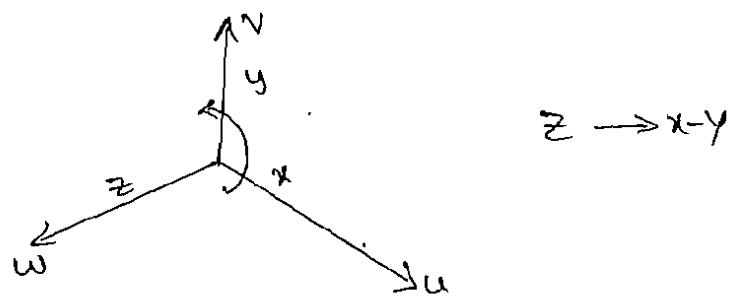
Rotational function properties:

A fluid element is said to be a rotation w.r.t  $z$ -axis

If its both axis horizontal ( $x$ -axis) as well as vertical



(y-axis) Rotates in the same direction x, y, z.



Conditions for Rotation:

The conditions for Rotation / Rotational components  
Rotation about z, x, y.

$$\omega_z = \frac{1}{2} \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$\omega_x = \frac{1}{2} \left[ \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right]$$

$$\omega_y = \frac{1}{2} \left[ \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right]$$

$$u = \frac{-\partial \phi}{\partial x}; \quad v = \frac{\partial \phi}{\partial y}; \quad w = \frac{-\partial \phi}{\partial z}$$

$$\omega_z = \frac{1}{2} \left[ -\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial x \partial y} \right]$$

$$\omega_x = \frac{1}{2} \left[ \frac{\partial^2 \phi}{\partial z \partial x} - \frac{\partial^2 \phi}{\partial x \partial z} \right]$$

$$\omega_y = \frac{1}{2} \left[ \frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial y \partial x} \right]$$

$$\begin{aligned} \omega_x &= 0 \\ \omega_y &= 0 \\ \omega_z &= 0 \end{aligned} \quad \left. \right\} \text{Irrational}$$

$\omega_x \neq \omega_y \neq \omega_z = 0 \rightarrow \text{Rotations}$

when the Rotational components are zero the flow is called irrotational. Hence the properties of the potential functions are.

x) If velocity potential Components are zero the flow is called irrotational. Hence, the properties of the potential functions are ] x

- 1) If velocity potential exists the flow should be irrotational.
- 2) If velocity potential Satisfies the laplace equation it represents the possible steady-incompressible irrotational flow.
- 3) If  $\phi = \text{Constant}$  at every point along the flow line is called equipotential line.

(or)

Along equipotential line the  $\phi$  is constant.

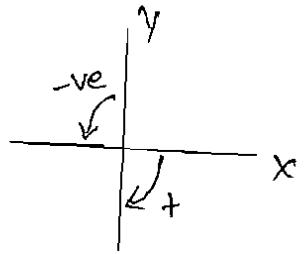
### Stream Function:

It is defined as the scalar function of space and time such that its partial derivative w.r.t any direction gives the velocity component at right angles to the direction. It is denoted by  $\psi$  (psi)

It is defined only for 2-dimensional flow so, Mathematically for Steady-flow it is defined as  $\psi(x,y)$  such that

$$\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = -u$$



For 2-dimensional flow the continuity eqn is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0 \implies 0=0$$

Hence existence of stream function  $\psi$  is a possible case of fluid flow. The flow may be Rotational or irrotational flow.

$$\omega_z = \frac{1}{2} \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$= \frac{1}{2} \left[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right]$$

$\omega_z \neq 0 \rightarrow$  rotational

$\omega_z = 0 \rightarrow$  irrotational

the properties of stream functions are:

- 1) If stream function  $\Psi$  exists it is a possible case of fluid flow which may be rotational or irrotational.
- 2) If stream function satisfies the Laplace equation It is a possible case of irrotational flow.

Relationship b/w velocity Potential function and Stream function:

velocity potential

$$u = -\frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \phi}{\partial y}$$

stream function

$$u = -\frac{\partial \Psi}{\partial y}$$

$$v = +\frac{\partial \Psi}{\partial x}$$

Relation

$$\frac{\partial \phi}{\partial x} = \frac{\partial \Psi}{\partial y}$$

$$-\frac{\partial \phi}{\partial y} = \frac{\partial \Psi}{\partial x}$$

Equi-potential line:

A line along which the velocity potential function is constant is known as Equi-potential line.

$$\phi \rightarrow \text{constant}$$

$$-\frac{\partial \phi}{\partial x} = u$$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$-\frac{\partial \phi}{\partial y} = v$$

$$d\phi = -u dx - v dy$$

$$\frac{\partial \phi}{\partial x} = -u ; \frac{\partial \phi}{\partial y} = -v$$

For equipotential line:

$$0 = -u \cdot dx - v \cdot dy$$

$$udx = -vdy$$

$$\therefore \frac{dy}{dx} = \frac{-u}{v}$$

The above equation represents the slope of equipotential line.

Stream line / Flow line:

A line along which stream line ( $\psi$ ) is constant is known as stream line / flow line.

$$\psi \rightarrow \text{Constant}$$

$$\frac{\partial \psi}{\partial x} = v$$

$$d\psi = 0$$

$$\frac{\partial \psi}{\partial y} = -u$$

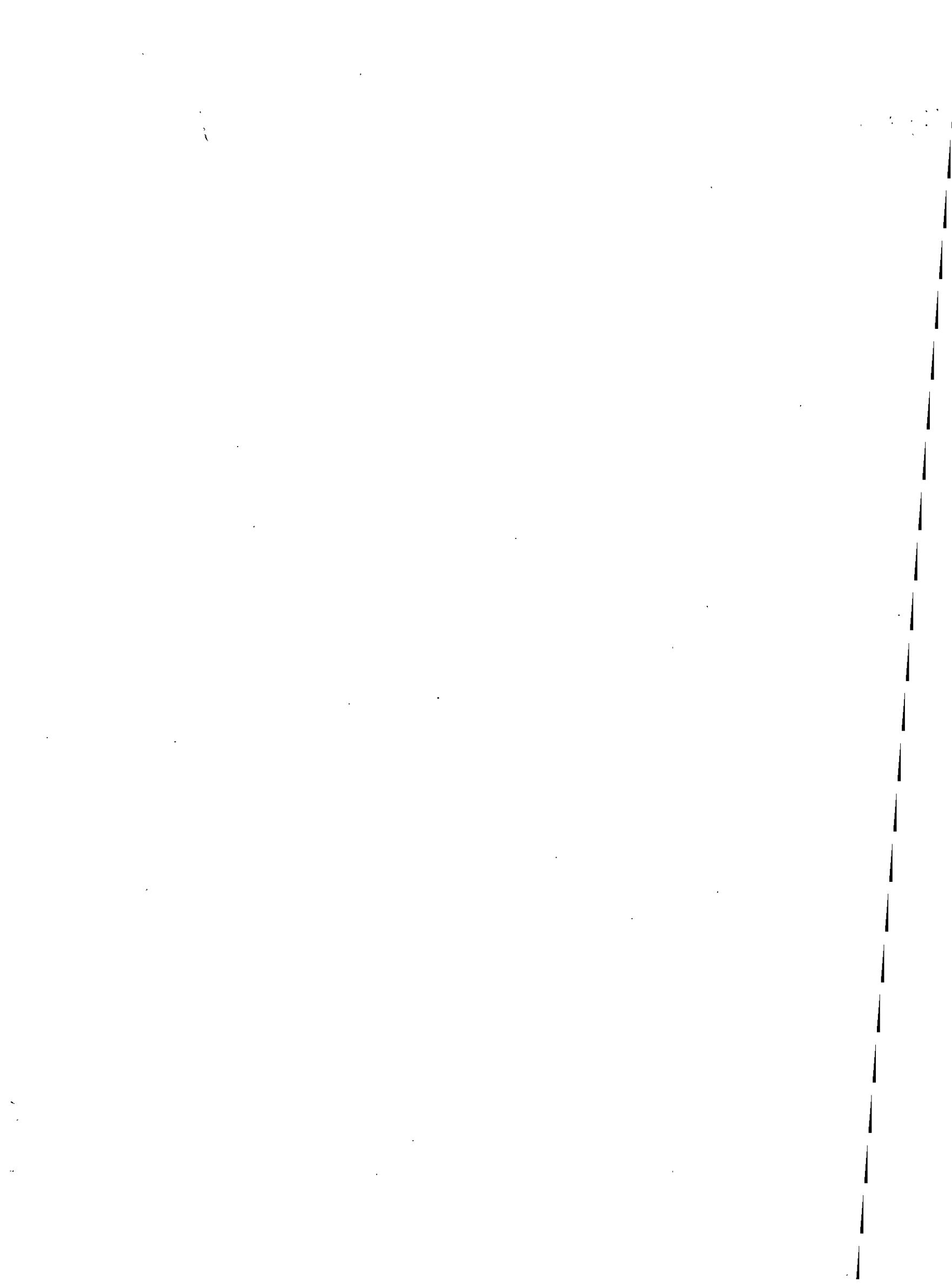
$$d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$$

$$0 = v \cdot dx - u \cdot dy$$

$$\frac{dy}{dx} = \frac{v}{u}$$

$\therefore$  The above equation represents the slope of stream line or flow line.

Note: The product of slope of equipotential line and straight line at the point of intersection is -1



### Problems

1) Is the flow represented by  $u=2x; v=-2y$  physically possible if so, obtain the expression for stream function. Is the flow is irrotational if so obtain velocity potential function.

Sol:

$$u=2x \quad : v=-2y$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$2-2=0$$

$$0=0 \text{ (True)}$$

$\therefore$  the flow is possible

$$\frac{\partial \psi}{\partial x} = -v$$

$$\frac{\partial \psi}{\partial x} = -2y$$

$$d\psi = -2y \cdot dx$$

$$\psi = -2xy + f(y)$$

As integration is done w.r.t "x" the constant term must be in terms of y or a numerical constant.

$$\frac{dy}{dx} = 2x + f'(y)$$

$$u = 2x + f'(y)$$

$$2u = 2x + f'(y)$$

$$f'(y) = 0 \Rightarrow f(y) = 0$$

$$\psi = -2xy + f(y) \underset{f(y)=0}{\approx} 0$$

$$\therefore \psi = -2xy$$

$\Leftarrow$  The above equation is expression for stream function

$$u_z = \frac{1}{2} \left[ \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right] = 0$$

$$u = 2x, v = -2y$$

$$\frac{\partial u}{\partial x} \geq 0, \quad \frac{\partial v}{\partial y} \geq 0$$

$$\frac{1}{2} (0 - 0) \geq 0$$

$$0 \geq 0 \quad (\text{True})$$

$\therefore$  The flow is irrotational.

$$\frac{\partial \phi}{\partial x} = -u$$

$$\frac{\partial \phi}{\partial x} = -2x$$

$$\int \partial \phi / \partial x = \int -2x \cdot dx$$

$$\phi = -\frac{2x^2}{2} + f(y)$$

$$\phi = -x^2 + f(y)$$

$$\frac{\partial \phi}{\partial y} = f'(y)$$

$$\frac{\partial \phi}{\partial y} = -v$$

$$-v = f'(y)$$

$$v = f'(y)$$

$$v^2 = f(y)$$

$$\phi = -x^2 + y^2$$

$$\therefore \phi = y^2 - x^2$$

- 2) The velocity components in a 2-D flow by an incompressible fluid are as follows  $u = \frac{y^3}{3} + 2x - xy^2$  and  $v = xy^2 - 2y - \frac{x^3}{3}$  obtain an expression for stream function  ~~$\psi$~~   $\psi$ .

Sol(?)

Given

$$u = \frac{y^3}{3} + 2x - xy^2$$

$$v = xy^2 - 2y - \frac{x^3}{3}$$

$$\frac{\partial v}{\partial x} = -v = \frac{\partial \psi}{\partial y} = u$$

$$\frac{\partial \psi}{\partial x} = -2 \left( xy^2 - 2y - \frac{x^3}{3} \right)$$

$$\frac{\partial \psi}{\partial x} = -xy^2 + 2y + \frac{x^3}{3} \rightarrow ①$$

$$\frac{\partial \psi}{\partial y} = \frac{y^3}{3} + 2x - x^2y \rightarrow ②$$

From ① &

$$\underline{\frac{\partial \psi}{\partial x}} = \left( -xy^2 + 2y + \frac{x^3}{3} \right) \cdot dx$$

$$\therefore \psi = -\frac{x^2y^2}{2} + 2xy + \frac{x^2}{12} + f(y)$$

$$\begin{aligned} \frac{\partial \psi}{\partial y} &= -\frac{x^2}{2} \cdot 2y + 2x + f'(y) \\ &= -x^2y + 2x + f'(y) \\ &= \left( -x^2y + 2x + \frac{y^3}{3} \right) \end{aligned}$$

$$\therefore f'(y) = \frac{y^3}{3}$$

$$f(y) = \frac{y^4}{12}$$

$$\therefore \psi = -\frac{x^2y^2}{2} + 2xy + \frac{x^4}{12} + \frac{y^4}{12}$$

3) The velocity components in a 2-D flow are  $u = \frac{y^3}{3} + 2x - x^2y$ :

$\omega = 2xy^2 - 2y - \frac{x^3}{3}$  show that these functions represents a possible case of irrotational flow

Sol)

$$\frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial y} = 0$$

$$2 - 2xy + 2xy - 2 = 0$$

$$0 = 0 \text{ (true)}$$

$\therefore$  The flow is possible

$$\frac{\partial u}{\partial z} = \frac{1}{2} \left[ \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right] = 0$$

$$= \frac{1}{2} [y^2 - x^2 - y^2 + x^2] = 0$$

$$0 = 0 \text{ (True)}$$

$\therefore$  The flow is irrotational.